

Modeling Longitudinal Dispersion in Premise Plumbing – is it needed?

Workshop on Mixing Processes
University of Sheffield

April 18, 2023

Steven Buchberger, PE, PhD

next
lives
here

University of
CINCINNATI

The logo of the University of Cincinnati, featuring a stylized red 'UC' monogram.

“It was 20 years ago today.....”

next
lives
here

“It was ~~20~~ 30 years ago today.....”

**next
lives
here**

MODELING CHLORINE RESIDUALS IN DRINKING-WATER DISTRIBUTION SYSTEMS

**By Lewis A. Rossman,¹ Member, ASCE, Robert M. Clark,²
Member, ASCE, and Walter M. Grayman,³ Member, ASCE**

Submitted April 15, 1993

Published July/August 1994

ASCE Journal of Environmental Engineering

**next
lives
here**

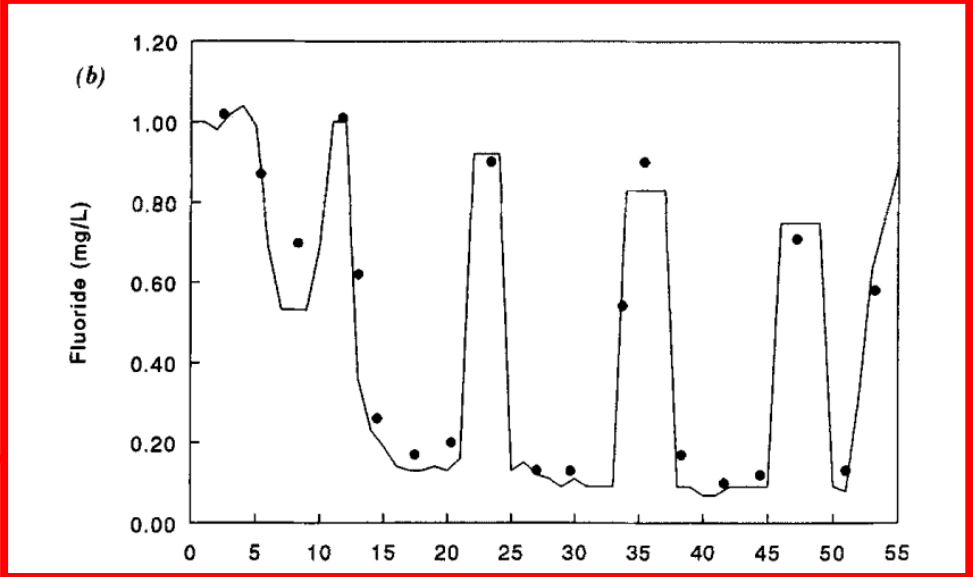
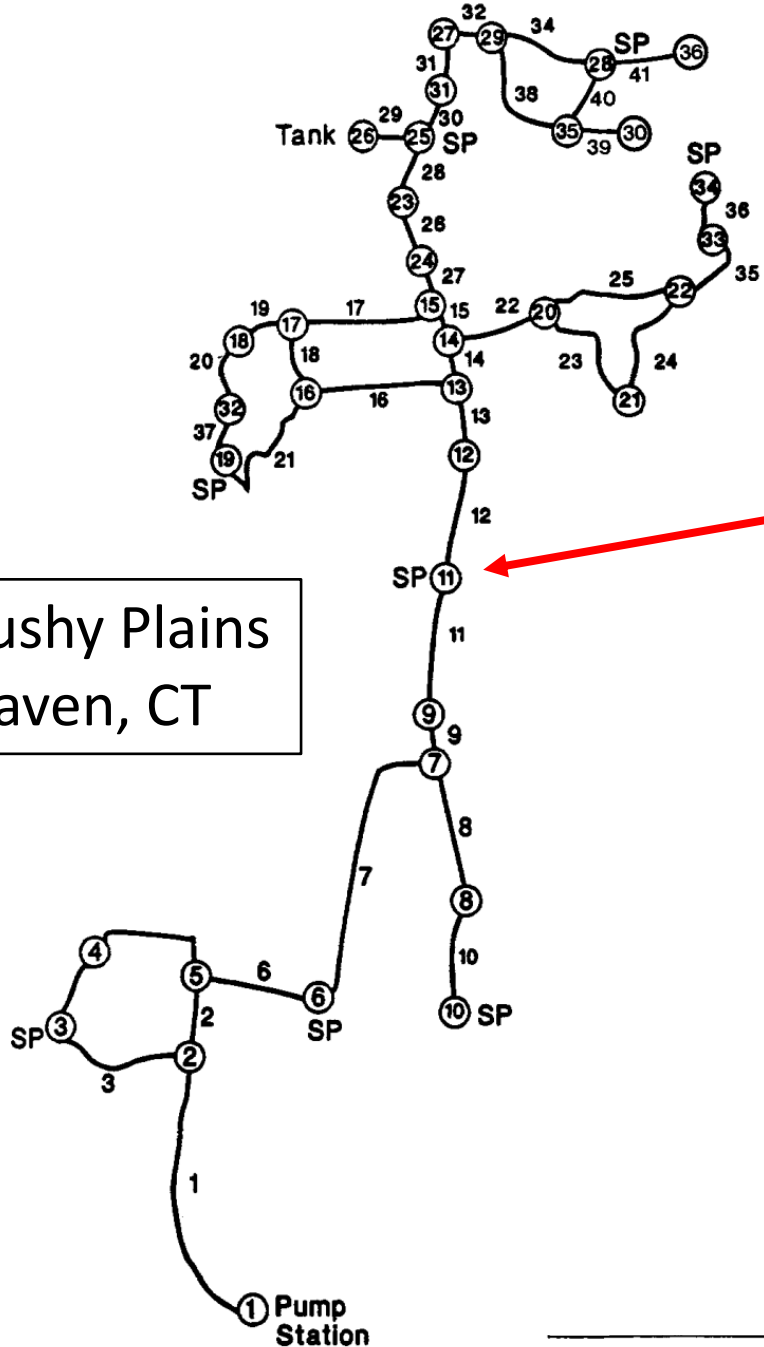
EPANET: Advection and Reaction

$$\frac{\partial c}{\partial t} = -u \frac{\partial c}{\partial x} - Kc$$

**next
lives
here**

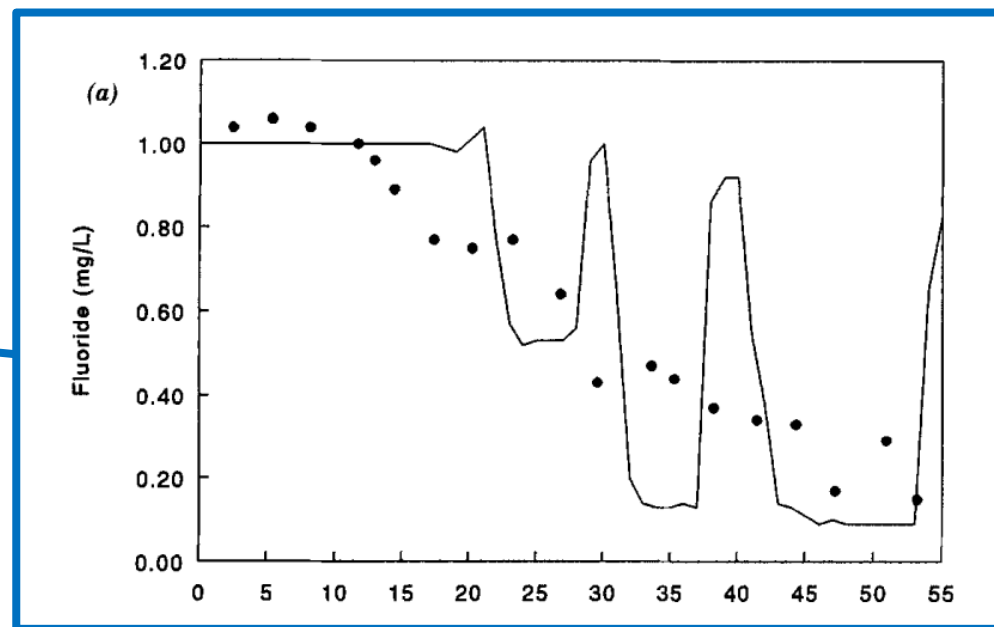
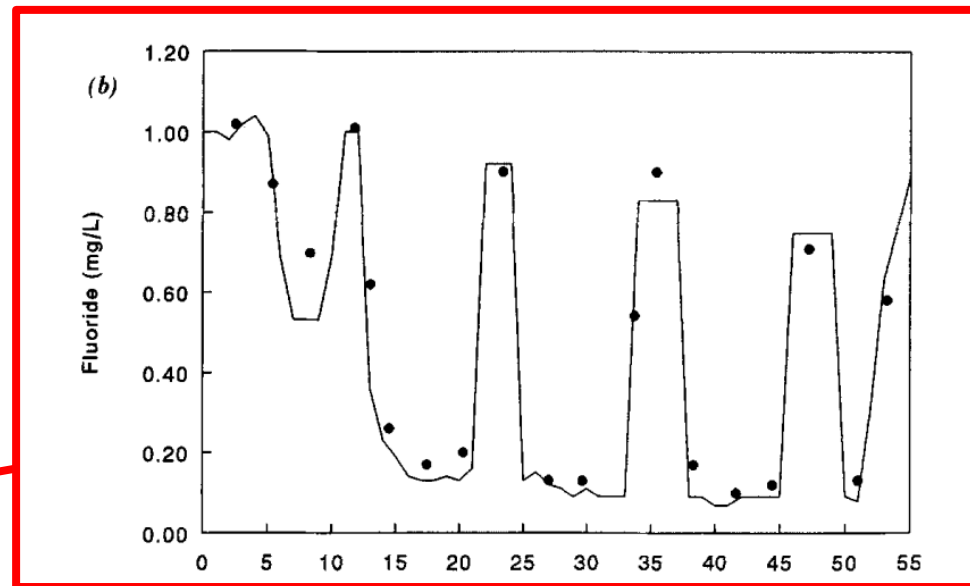
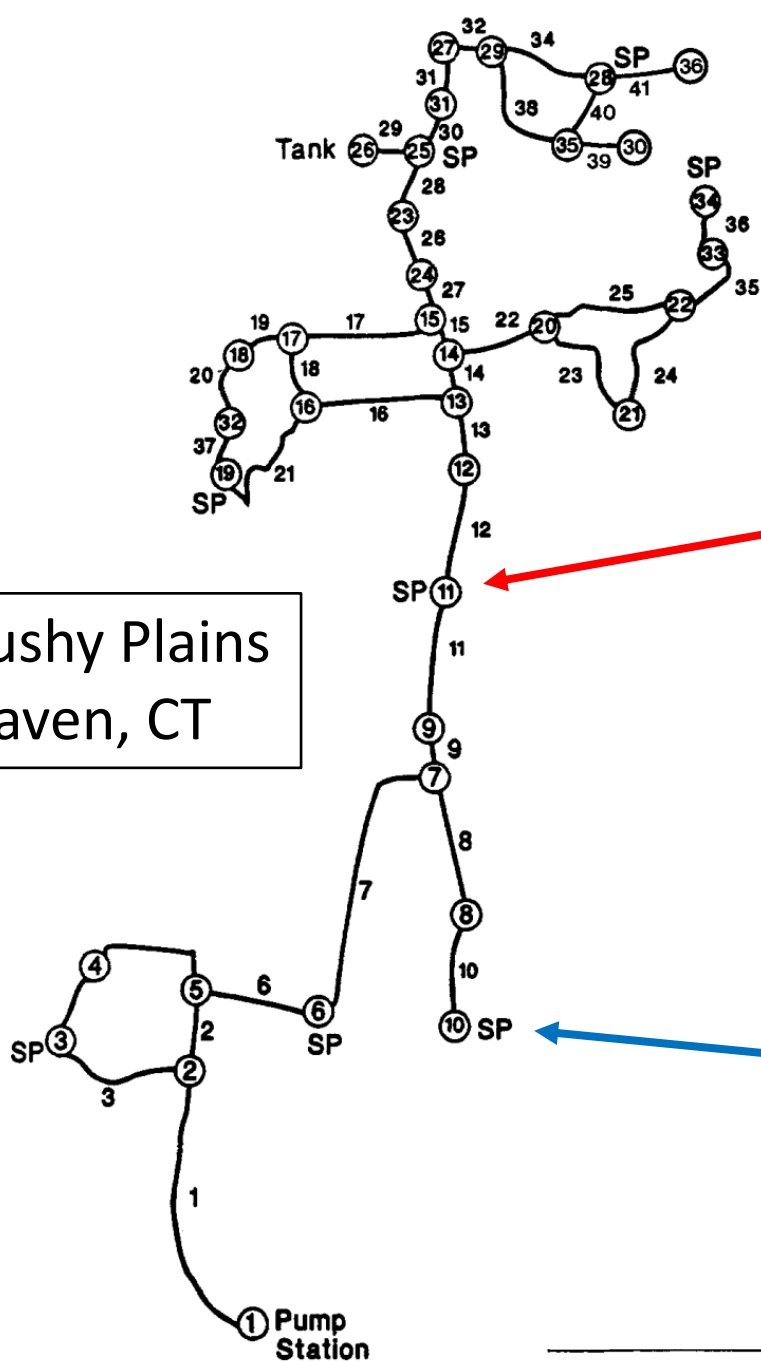
Cherry Hills Brushy Plains
WDS in New Haven, CT

next
lives
here



Cherry Hills Brushy Plains WDS in New Haven, CT

next
lives
here



Water Quality Focus Launched New Era in Water Demand Modeling

“Model predictions were less accurate at sites where the hydraulic calibration was less successful. These results underscore the need to obtain accurate hydraulic information before running a network water-quality model.”

Rossman, Clark and Grayman (1994)

MODEL FOR INSTANTANEOUS RESIDENTIAL WATER DEMANDS

By Steven G. Buchberger,¹ Member, ASCE, and Lin Wu²

Submitted September 1993

Published March 1995

ASCE Journal of Hydraulic Engineering

**next
lives
here**

PRP: Poisson Rectangular Pulse Model for Residential Water Demands

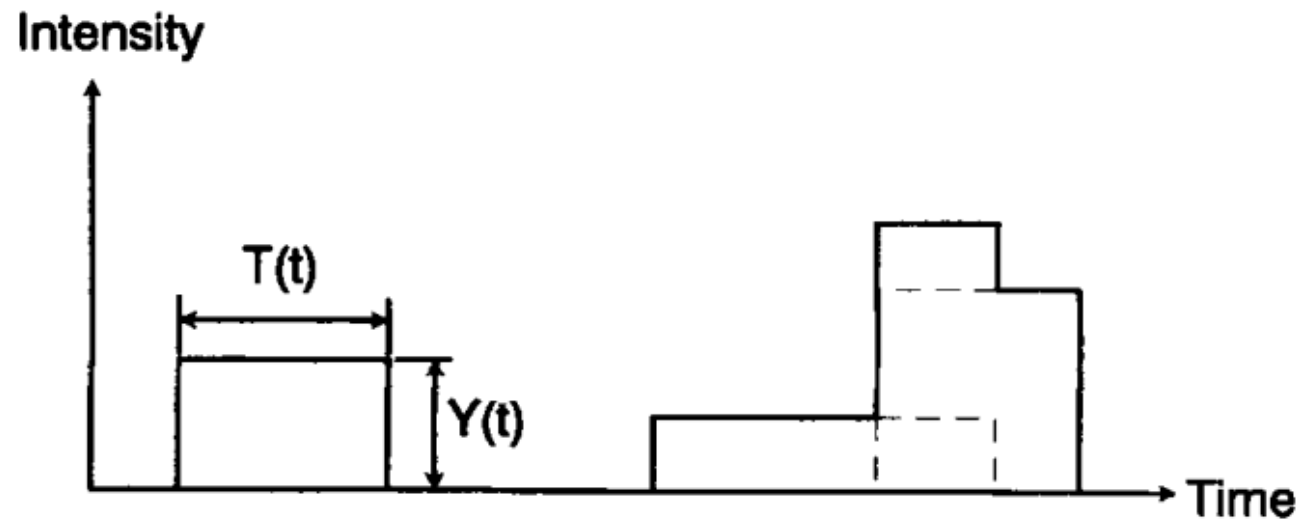
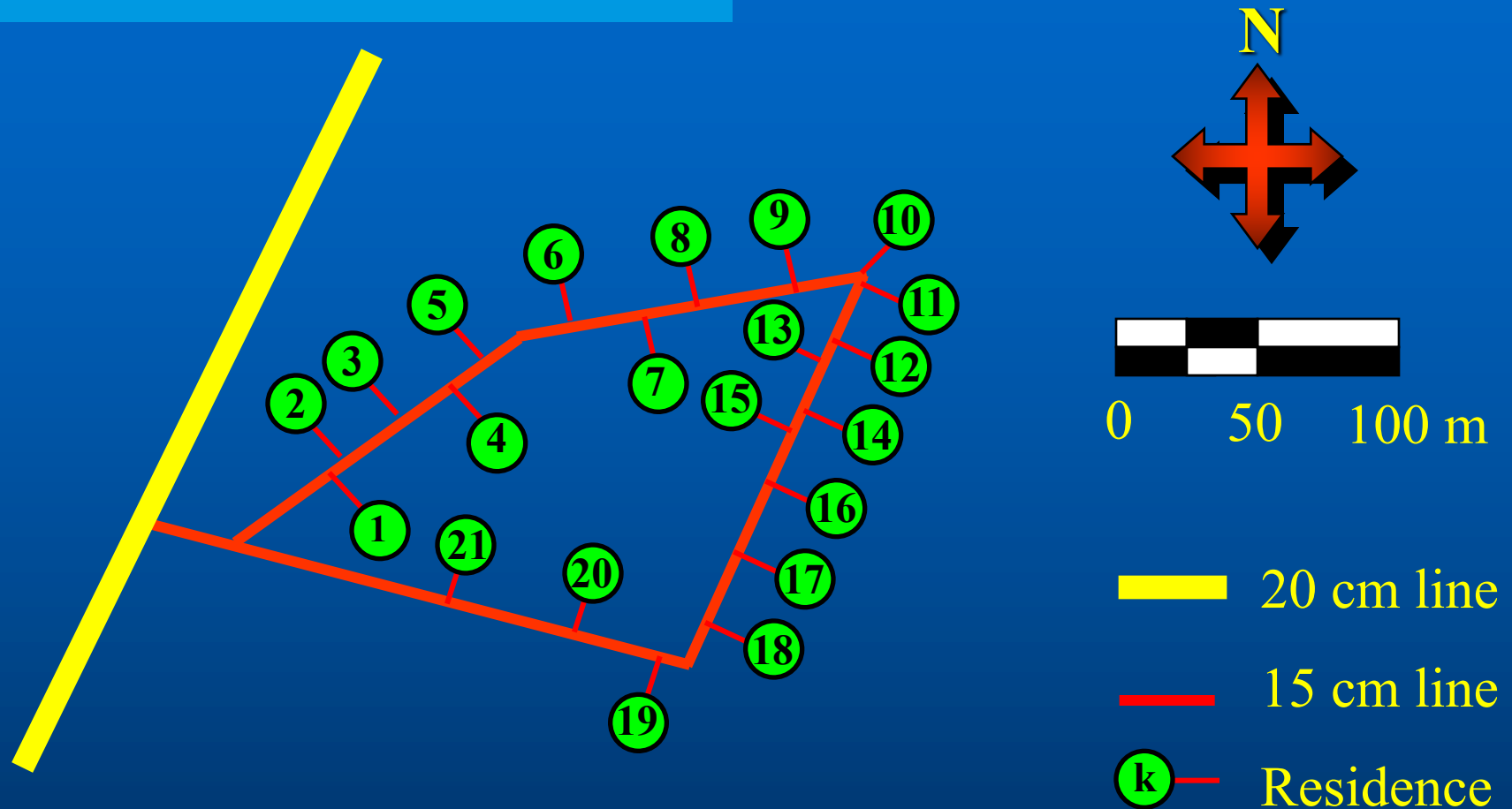


FIG. 2. Rectangular Pulses Used to Represent Water Demands at Residential Servers

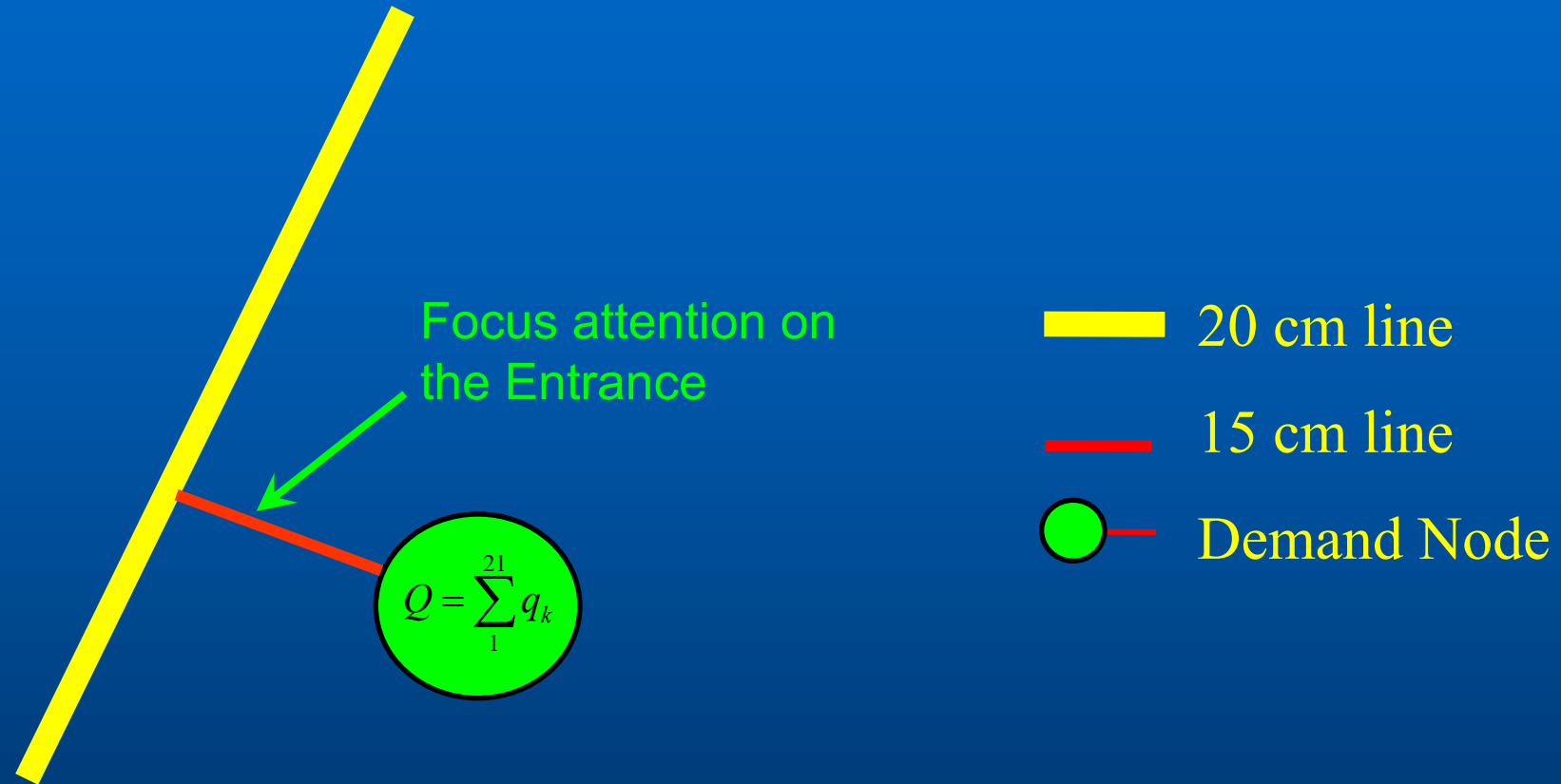
Buchberger and Wu (1995)

next
lives
here

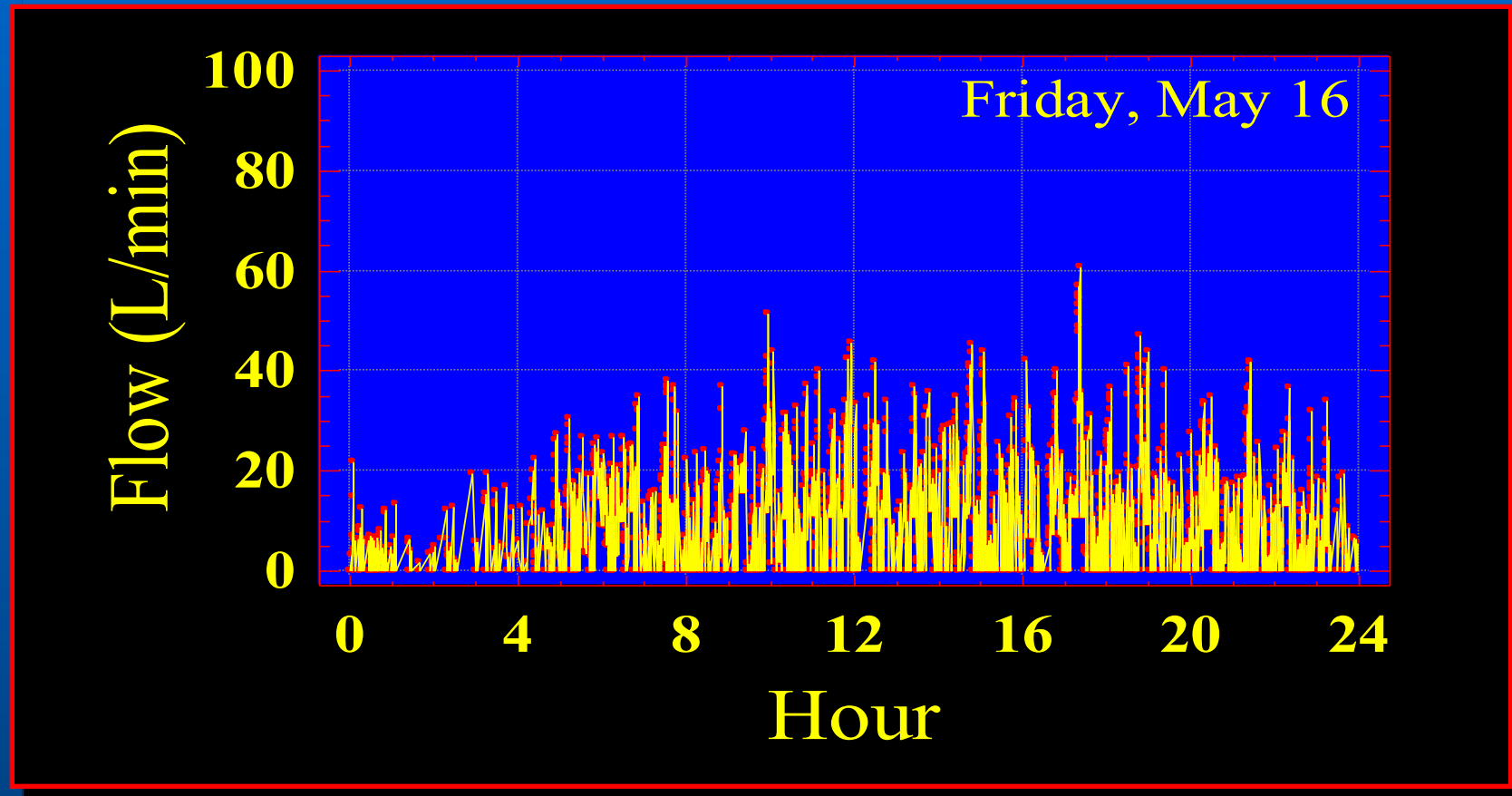
Milford Study Site



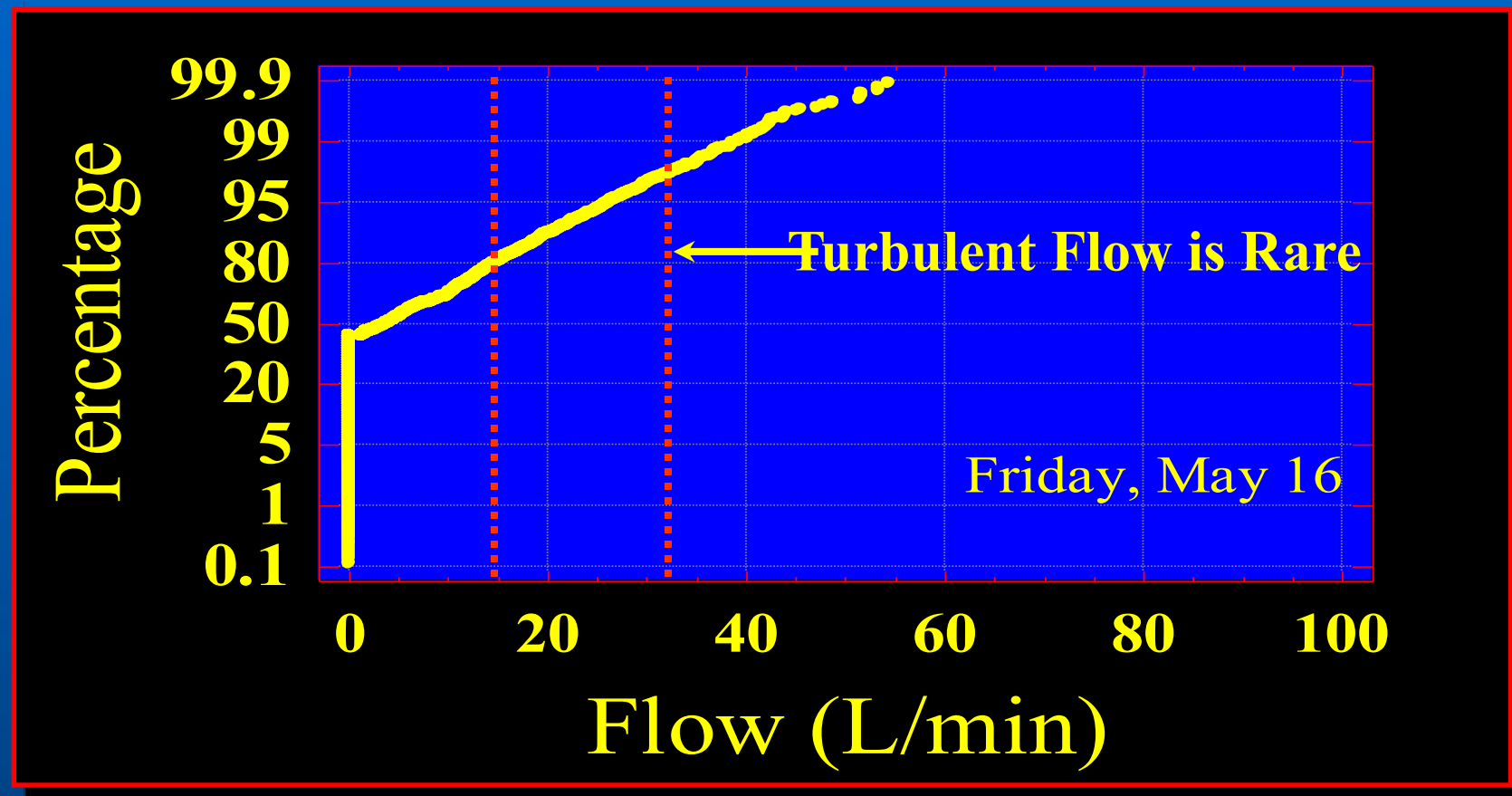
Consolidate 21 Homes into 1 Node



Demand Hydrograph at Entrance



Probability Plot for Flows at Entrance



Advection-Dispersion-Reaction Modeling in Water Distribution Networks

Velitchko G. Tzatchkov¹; Alvaro A. Aldama²; and Felipe I. Arreguin³

Submitted December 1999

Published September 2002

ASCE Journal of Water Resources Planning & Management

**next
lives
here**

IMTARED Introduced 1-D Dispersion to Water Distribution System Models

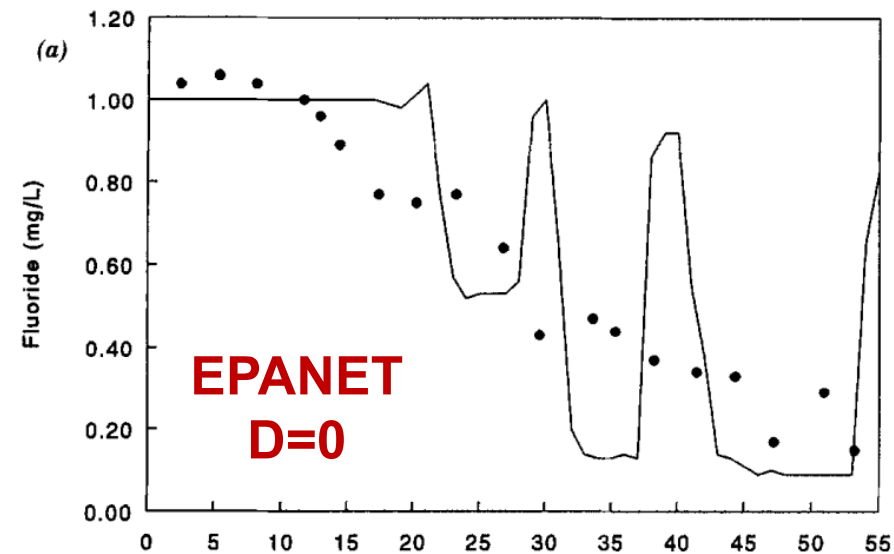
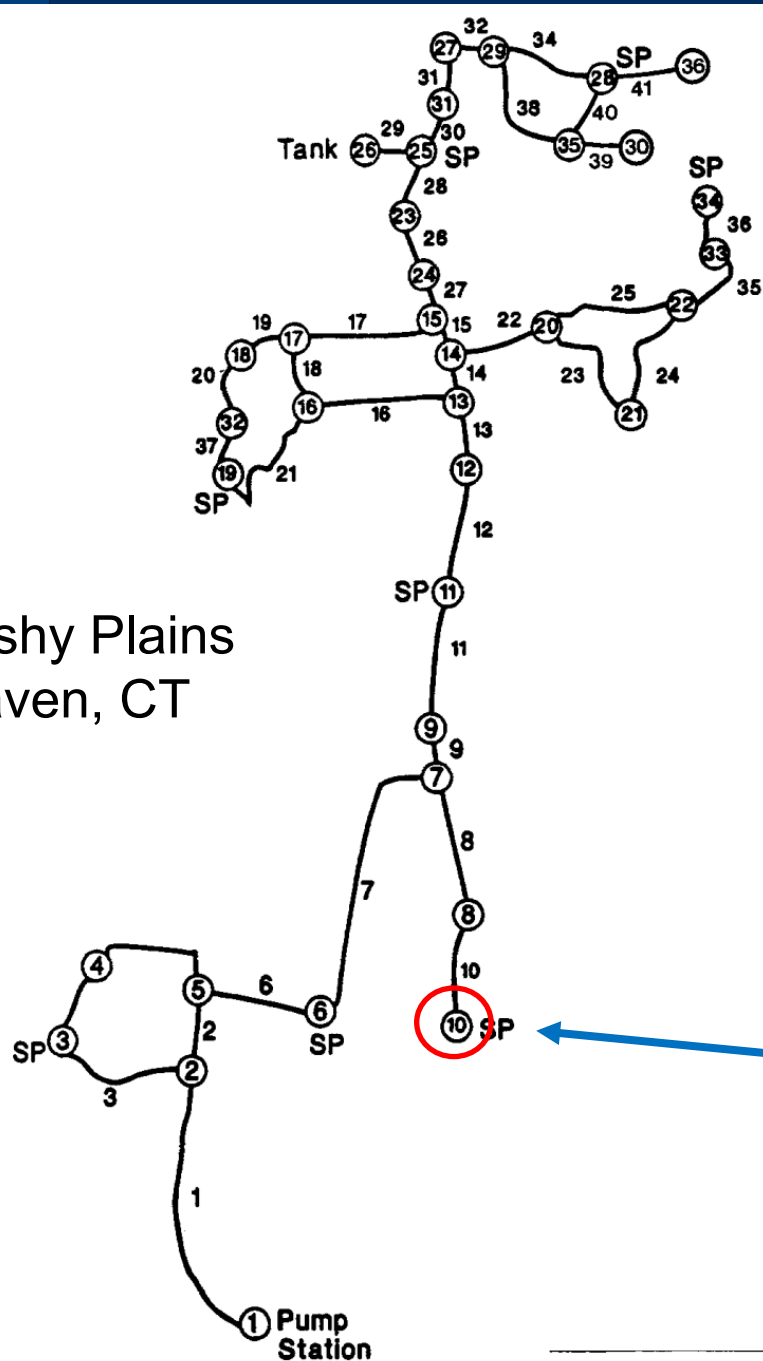
$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x} - Kc$$

next
lives
here

Tzatchkov, Aldama, Arreguin (2002)

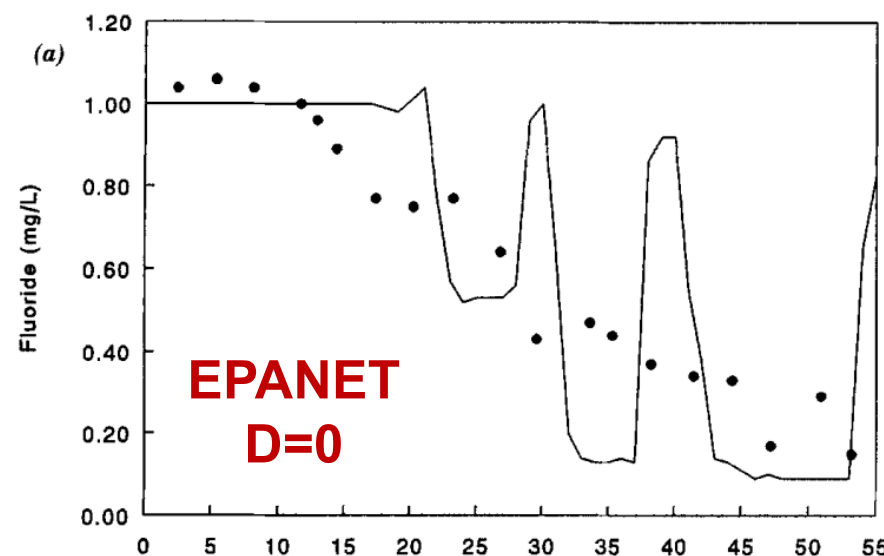
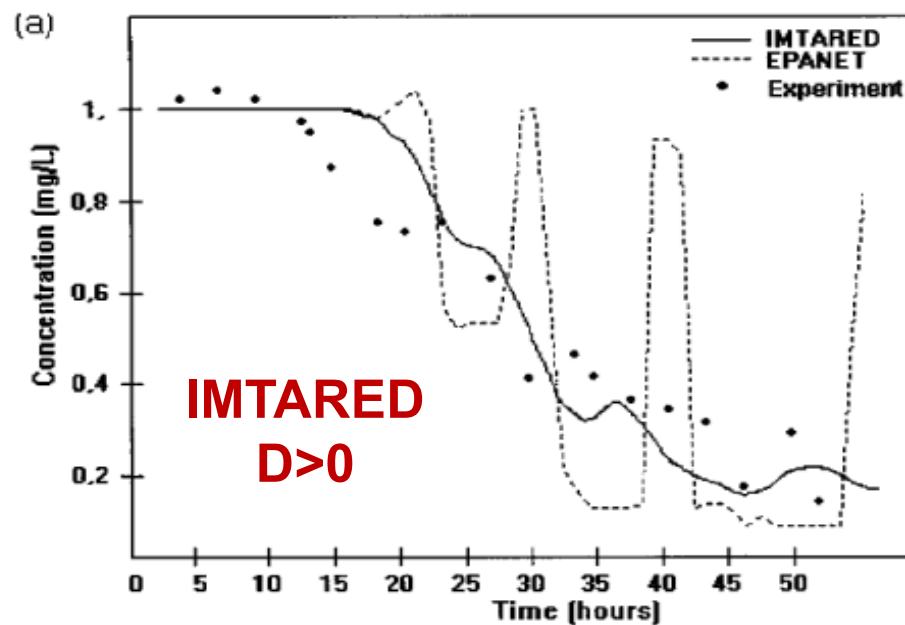
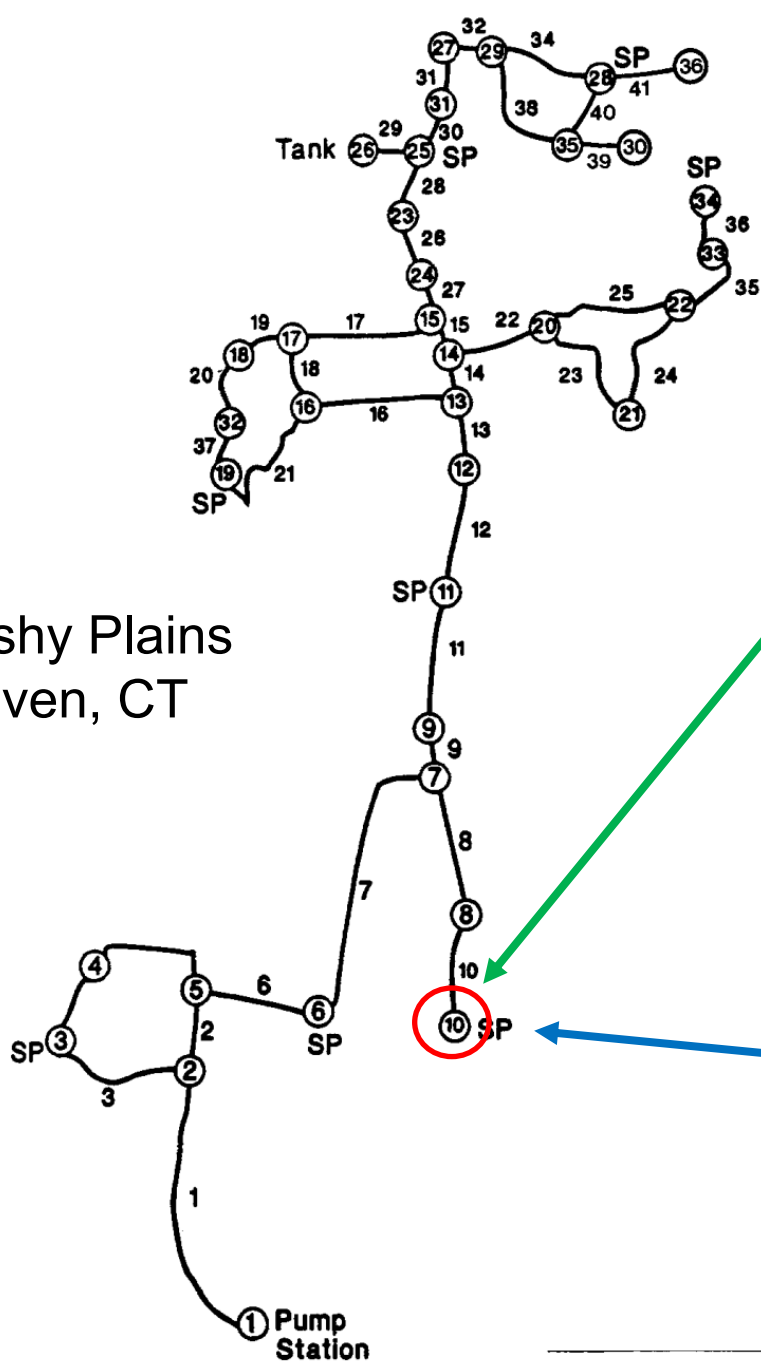
Cherry Hills Brushy Plains WDS in New Haven, CT

next
lives
here



Cherry Hills Brushy Plains
WDS in New Haven, CT

next
lives
here



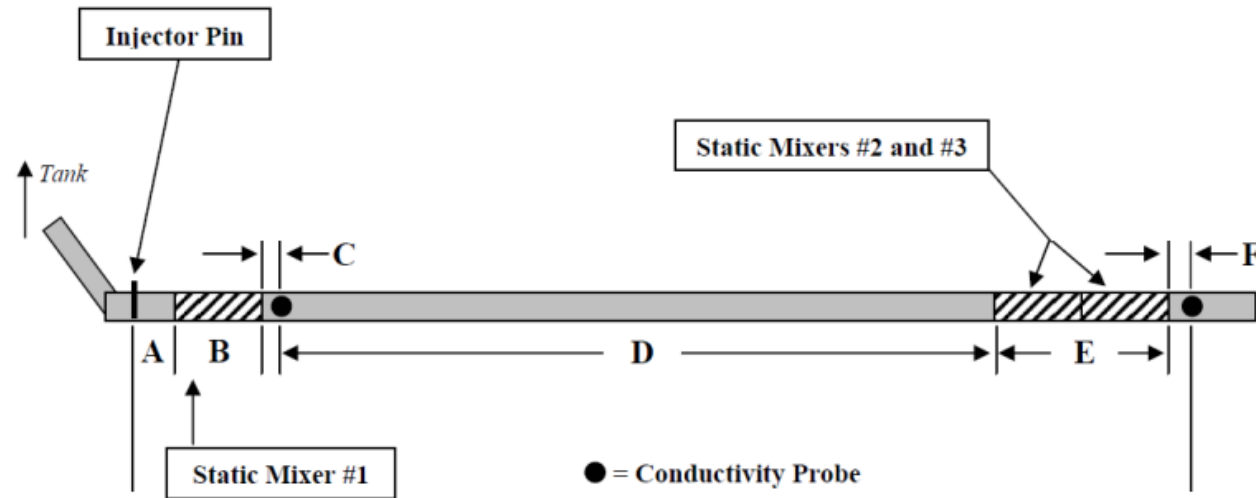
Predicting Rates of Laminar Dispersion

Three Expressions for Dispersion Coefficient in Laminar Pipe Flow	Reference	Applicability
$D_T = \frac{a^2 U^2}{48 D_m}$	Taylor (1953)	$T = \frac{D_m t}{a^2} \geq 0.5$
$D_{GS}(T) = D_T \left[1 - 768 \sum_{n=1}^{\infty} \left(\frac{J_3(\lambda_n) J_2(\lambda_n)}{\lambda_n^5 [J_0(\lambda_n)]^2} \right) \exp(-\lambda_n^2 T) \right]$	Gill and Sankarasubramanian (1970)	$T \geq 0$
$D_L(T) = D_T [1 - \exp(-16T)]$	Lee (2004)	$T \geq 0$

**next
lives
here**

There is 99.99% correlation between Lee (2004) and Gill/Sankarasubramanian (1970) predictions

Experimental Facility for Measuring Dispersion Rates in Laminar Flow

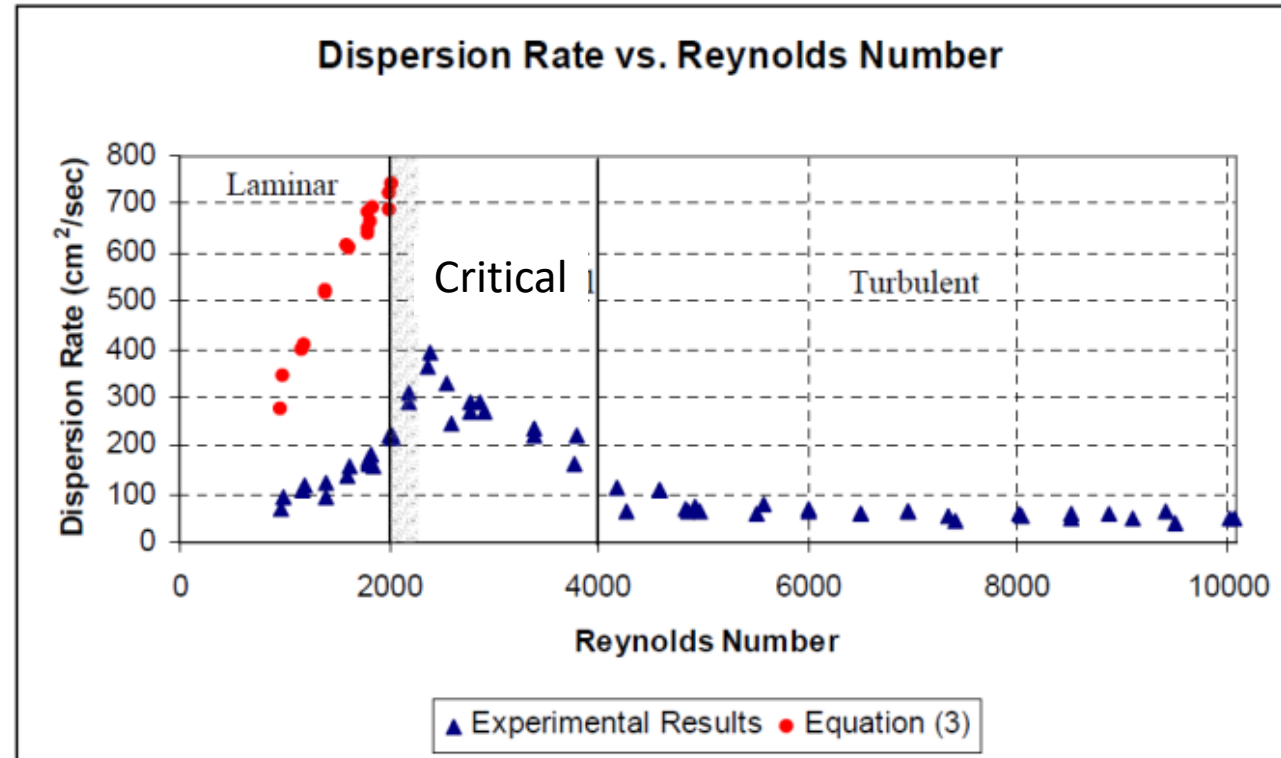


Source: Cutter (2004).

Units	A	B	C	D	E	F	Total
English (inches)	6.125	55.0	17.0	1534.2	110.0	17.0	1739.3
SI (centimeters)	15.6	139.7	43.2	3896.9	279.4	43.2	4418.0
Component	Injection	Mixer #1	Probe #1	Pipe	Mixers 2&3	Probe #2	
Volume (L)	2.57	20.1	0.386	641.5	40.2	7.1	712.4

next
lives
here

Predicted and Observed Dispersion Rates in Laminar Flow through Circular Pipe



Experimental Results are Based on Hugo Fischer's Method of Moments

Predicted Results are Based on Lee (2004).

Source: Cutter (2004).

Taylor's **Laminar** D = 20,000 to 120,000 cm²/sec

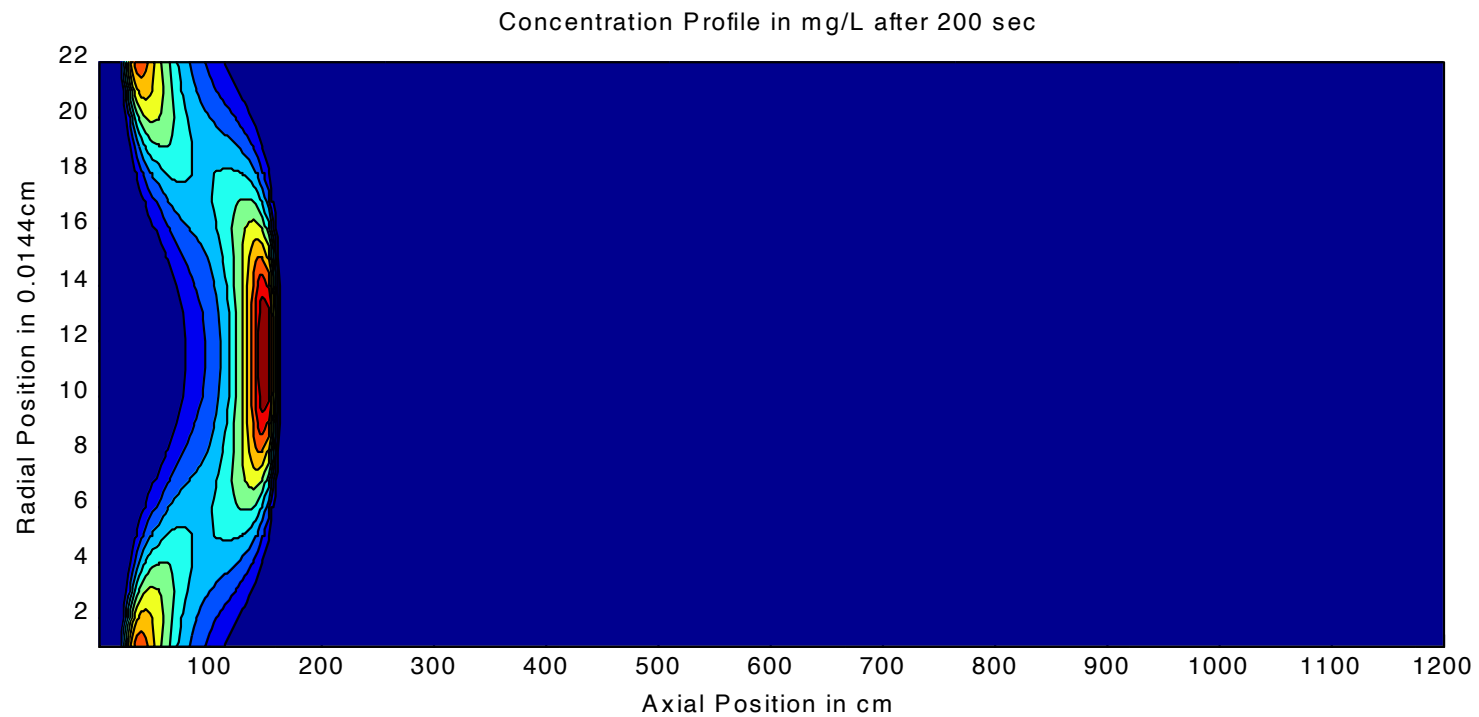
next
lives
here

Numerical Simulation of Mass Pulse Transported in Intermittent Laminar Flow through a Circular Pipe

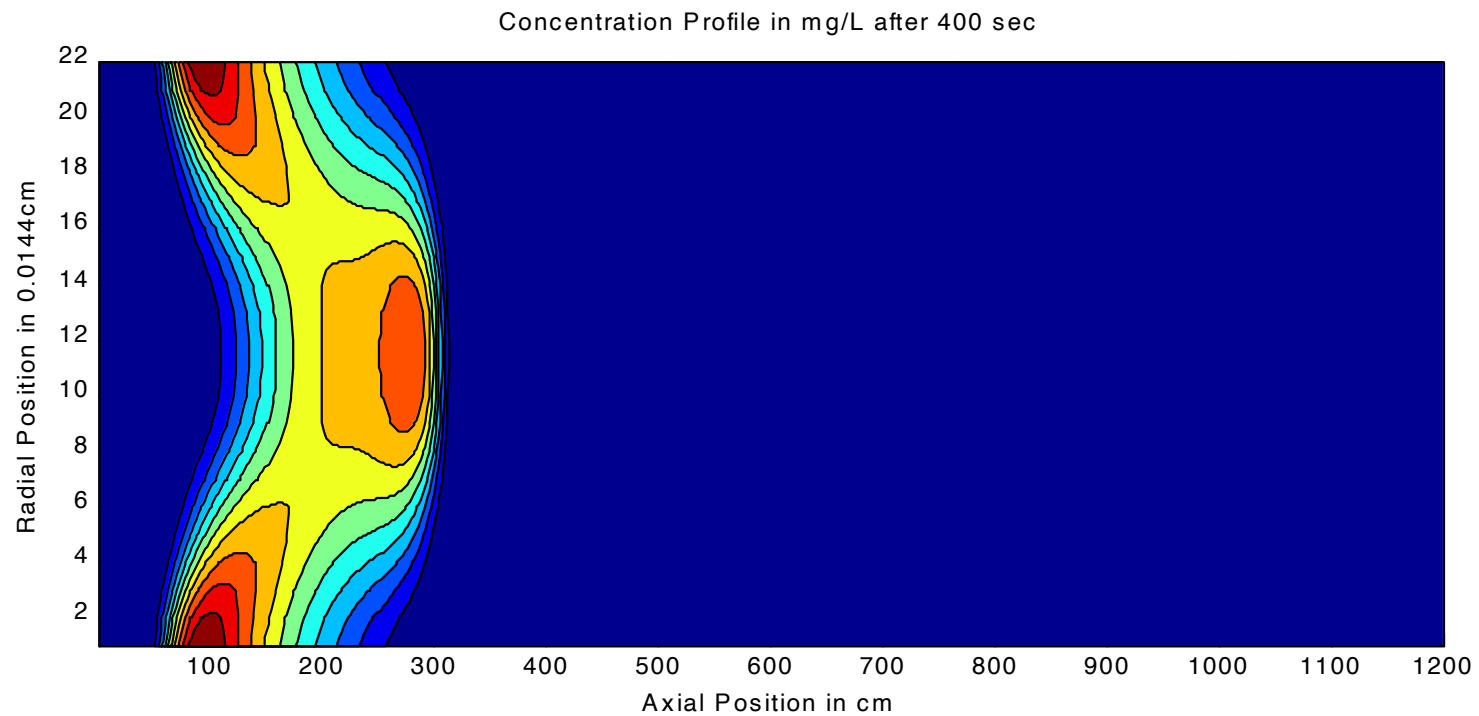
(13 slides in rapid succession)
(vertical exaggeration 3800:1)

next
lives
here

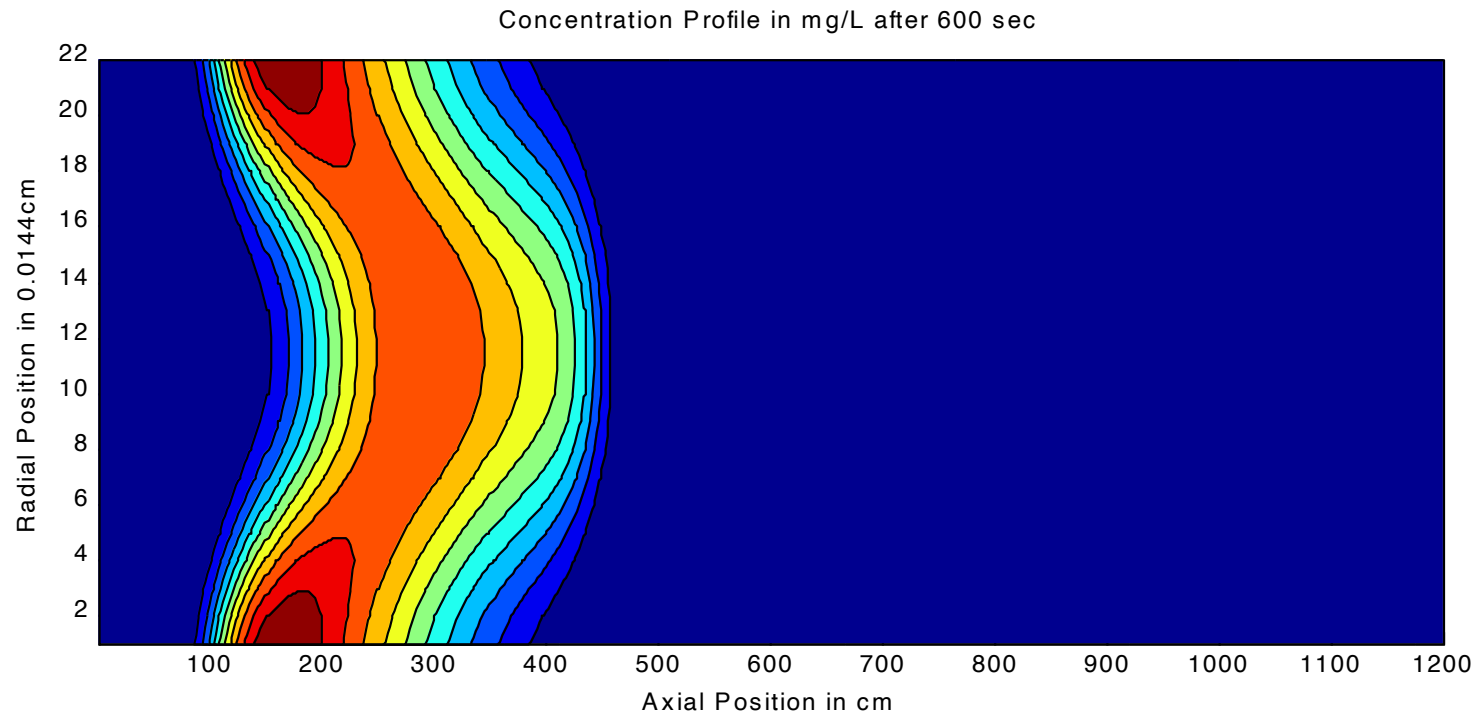
Dispersion in Laminar Flow, $T=0.1$



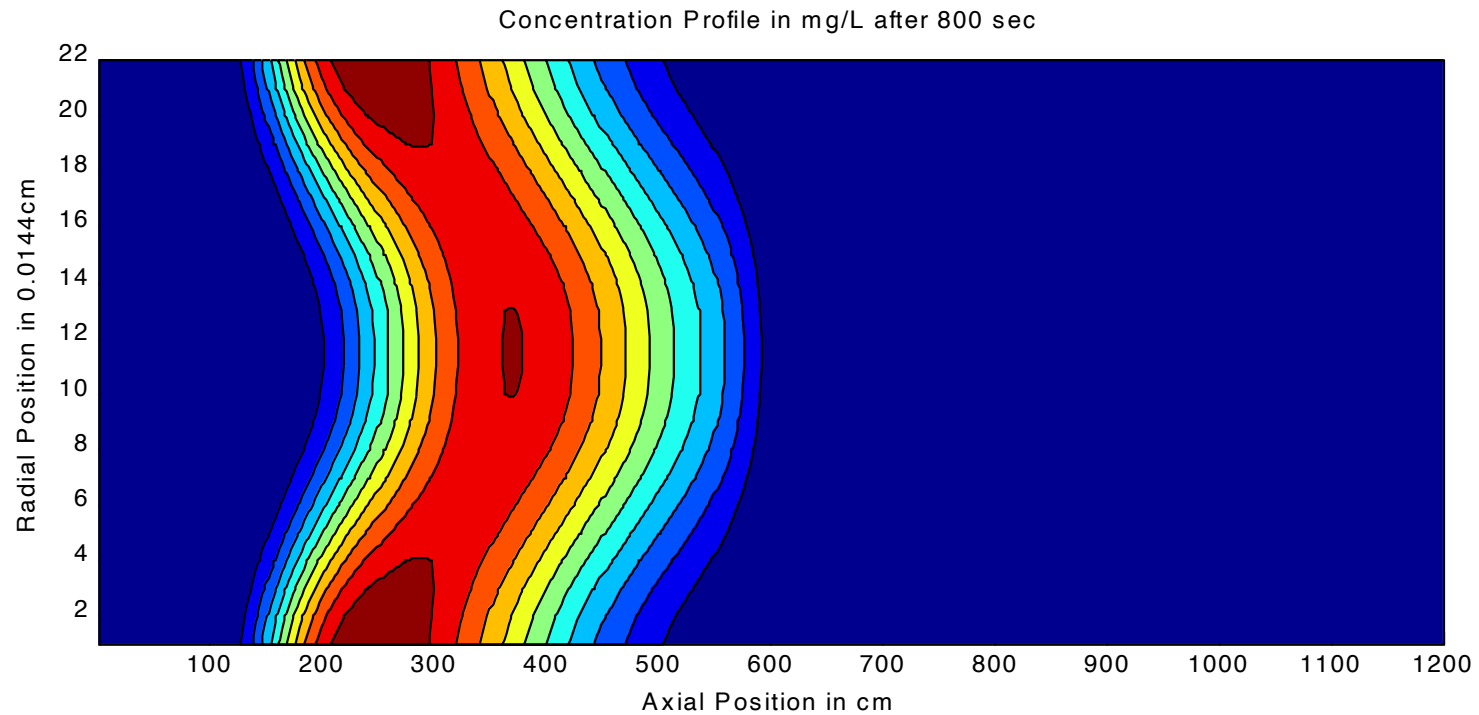
Dispersion in Laminar Flow, $T=0.2$



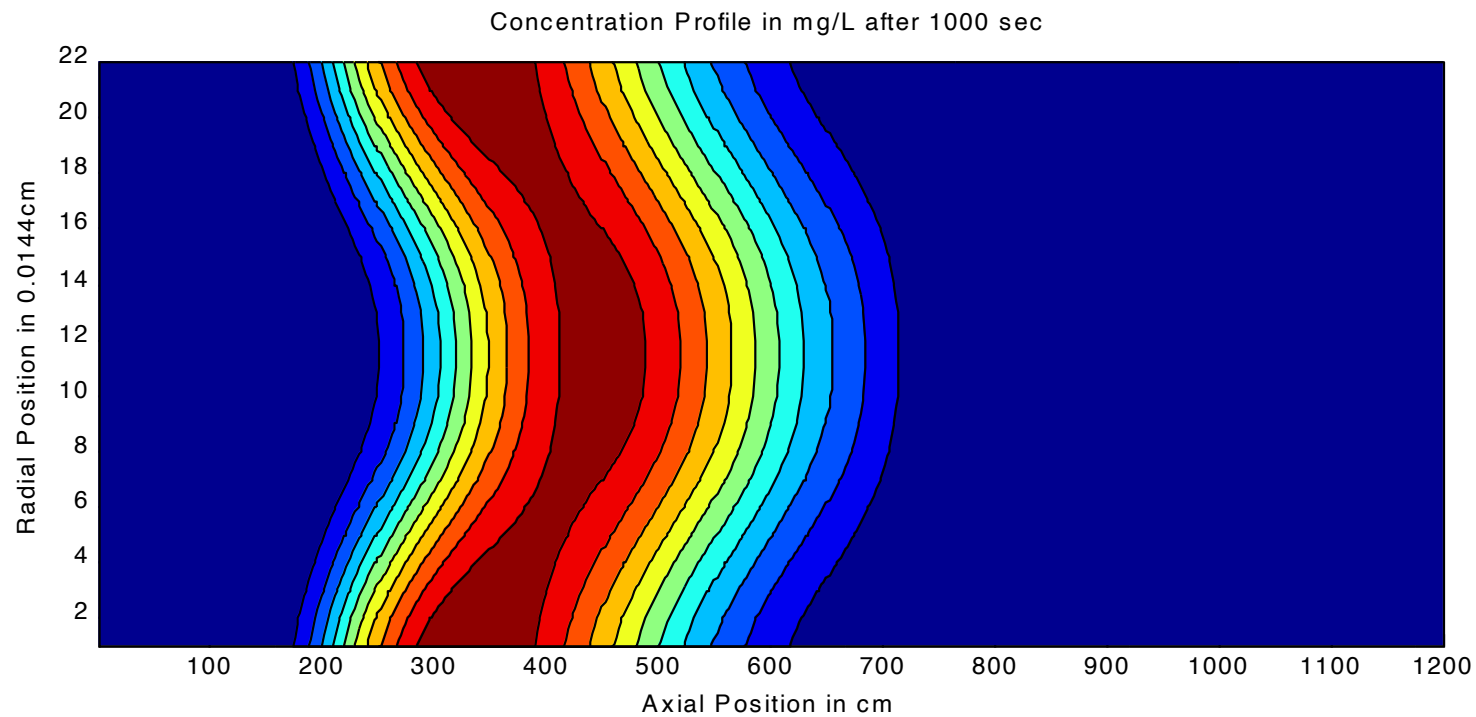
Dispersion in Laminar Flow, $T=0.3$



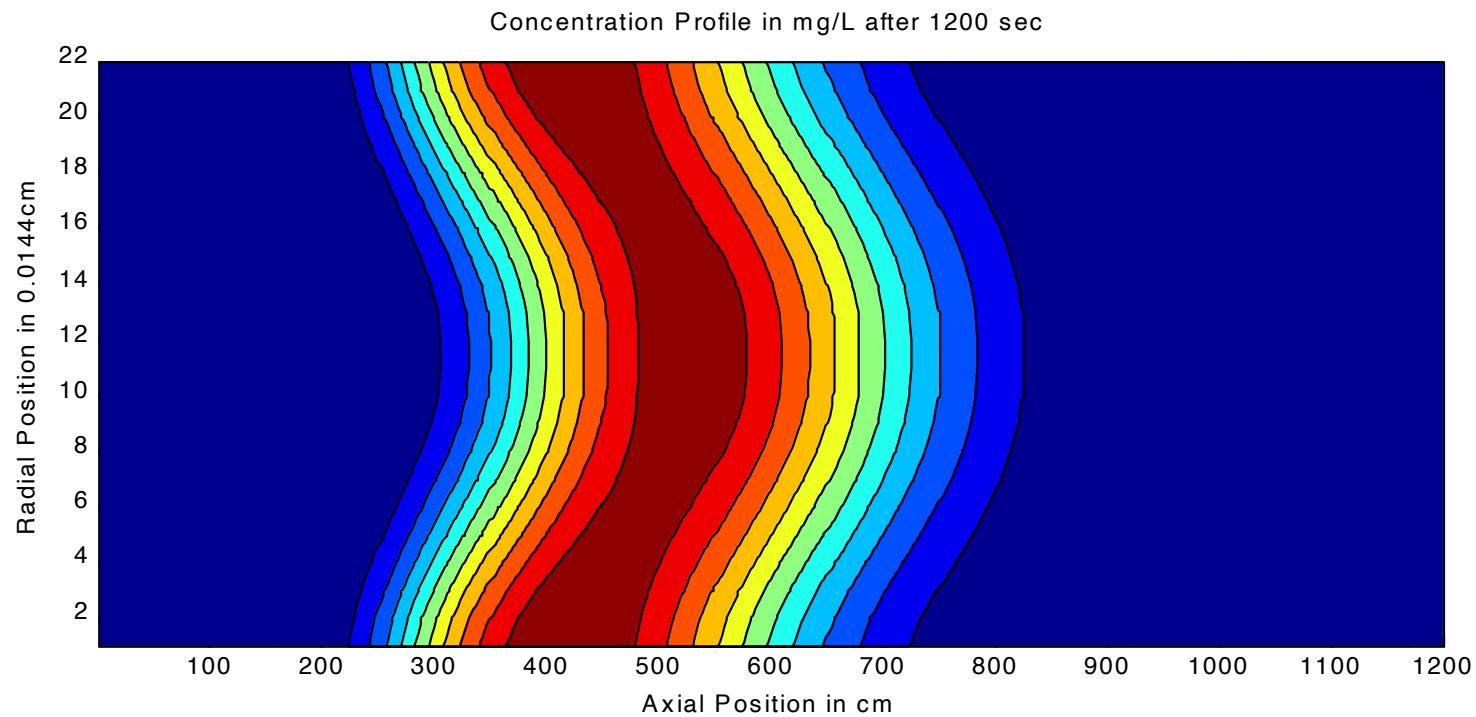
Dispersion in Laminar Flow, $T=0.4$



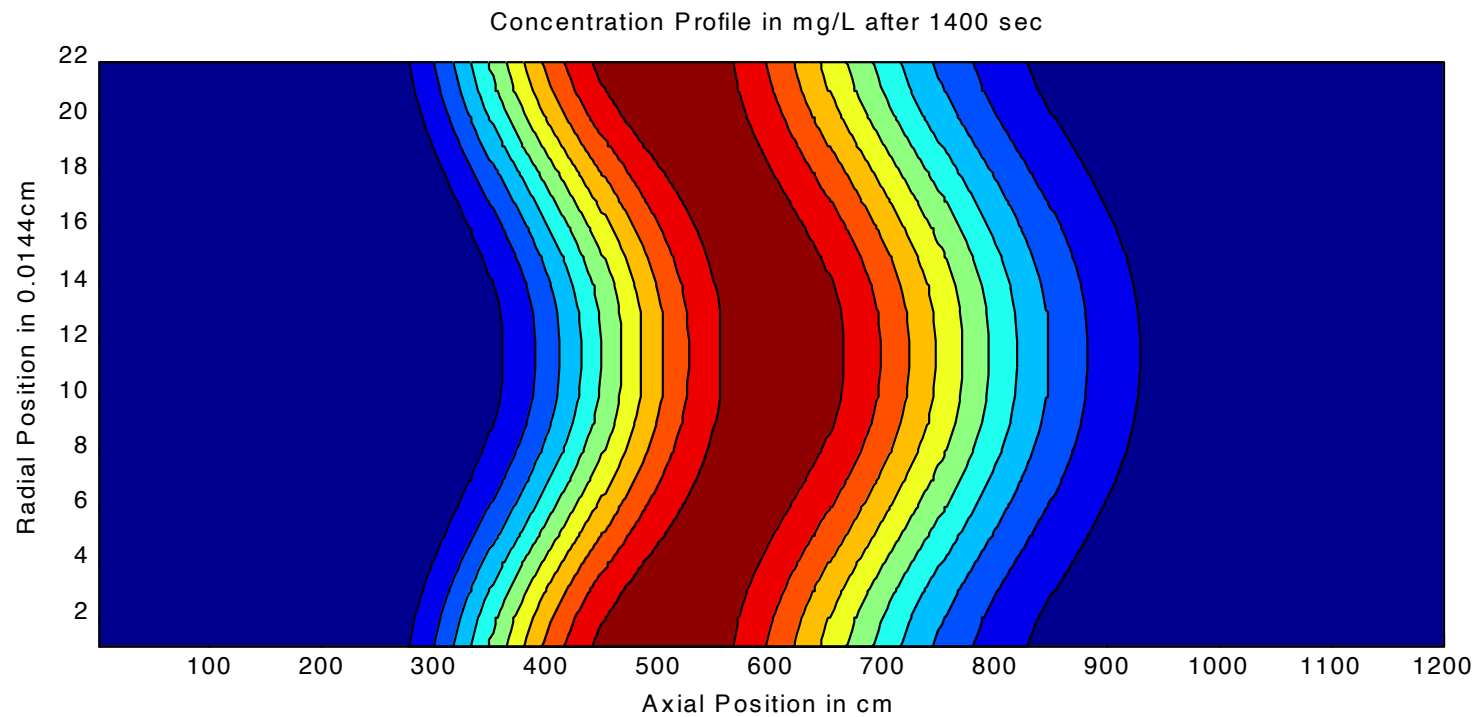
Dispersion in Laminar Flow, $T=0.5$



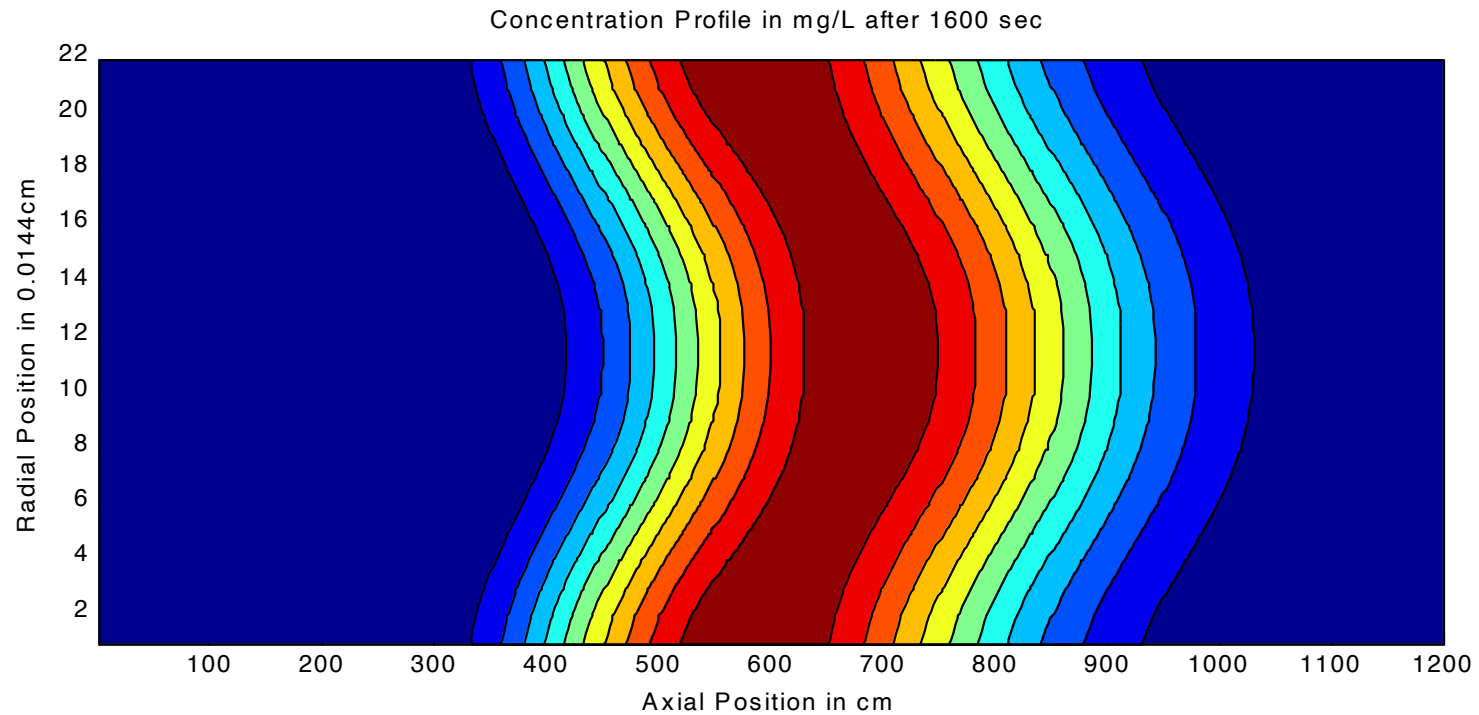
Dispersion in Laminar Flow, $T=0.6$



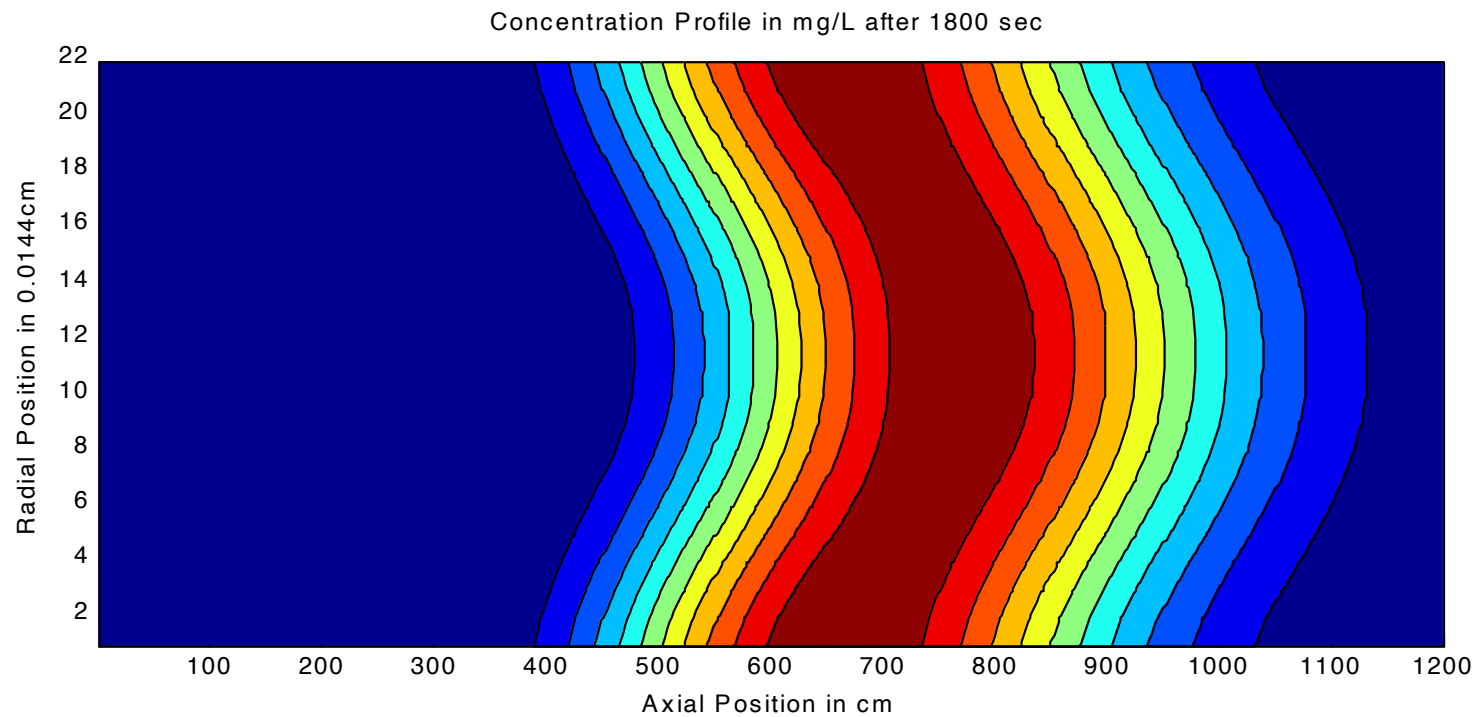
Dispersion in Laminar Flow, $T=0.7$



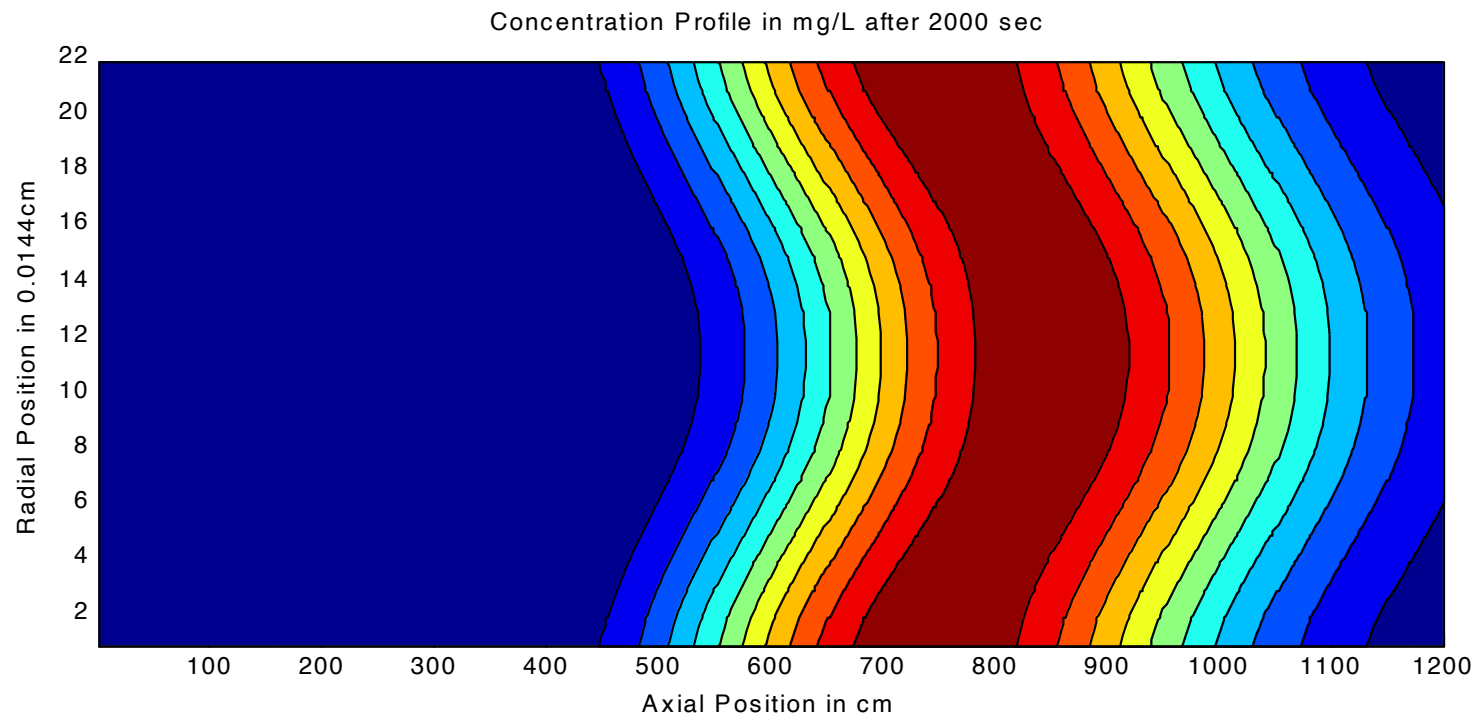
Dispersion in Laminar Flow, $T=0.8$



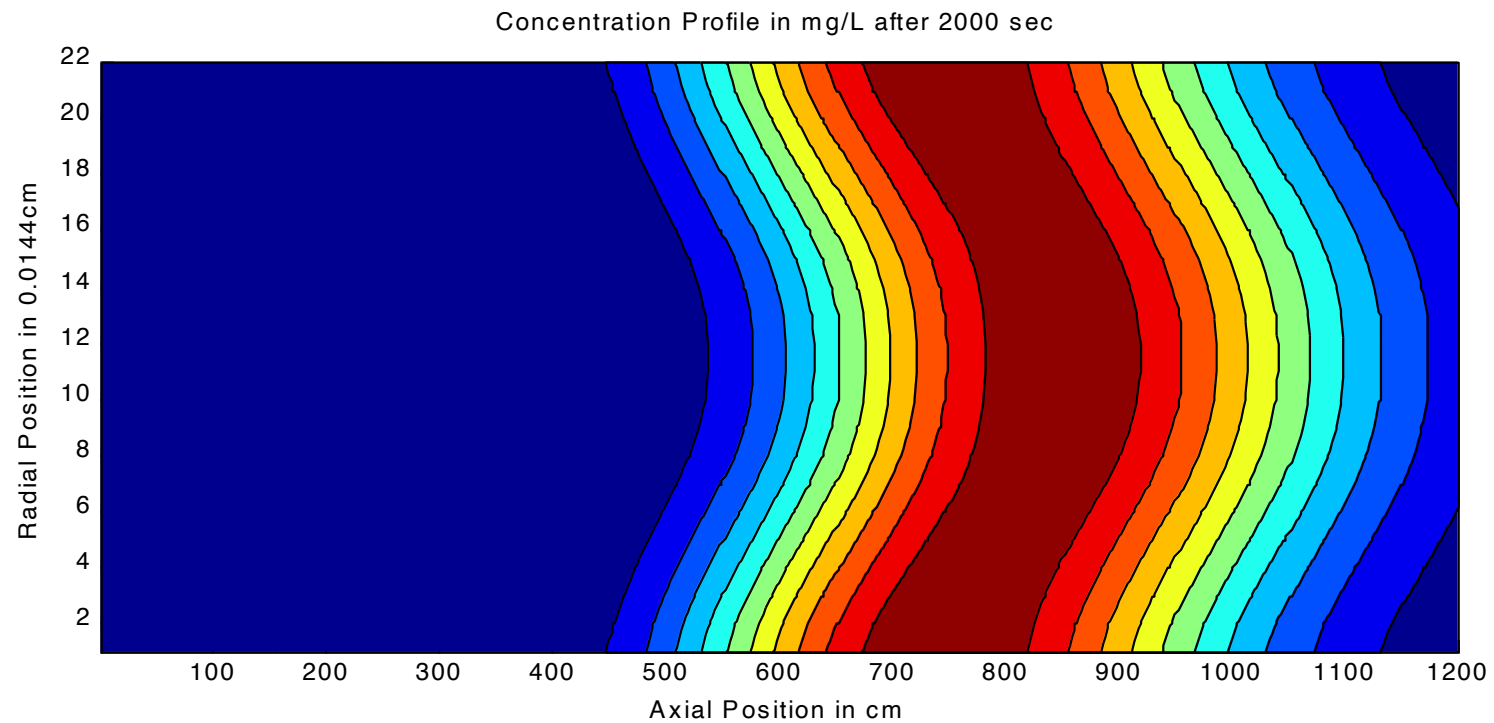
Dispersion in Laminar Flow, $T=0.9$



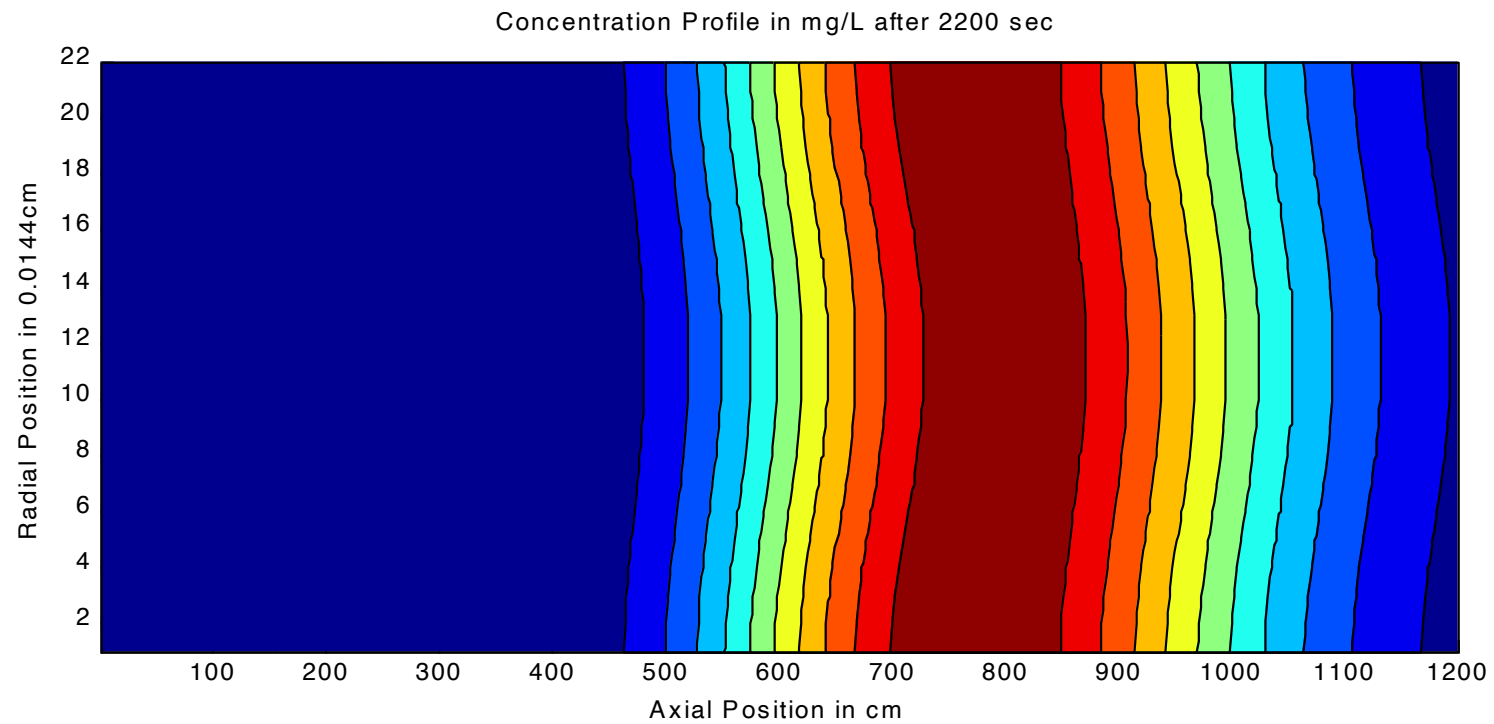
Dispersion in Laminar Flow, $T=1.0$



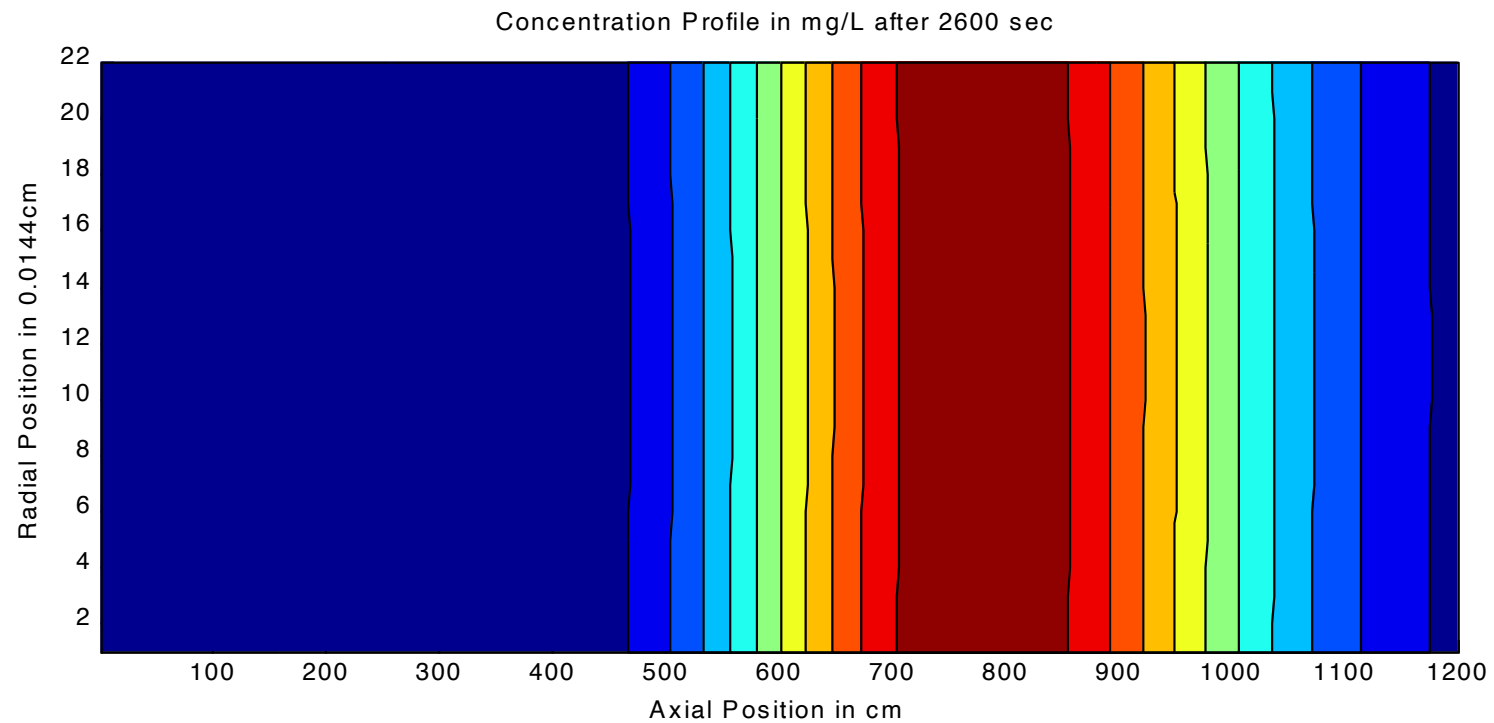
Relaxation in Laminar Flow, $T=0.0$



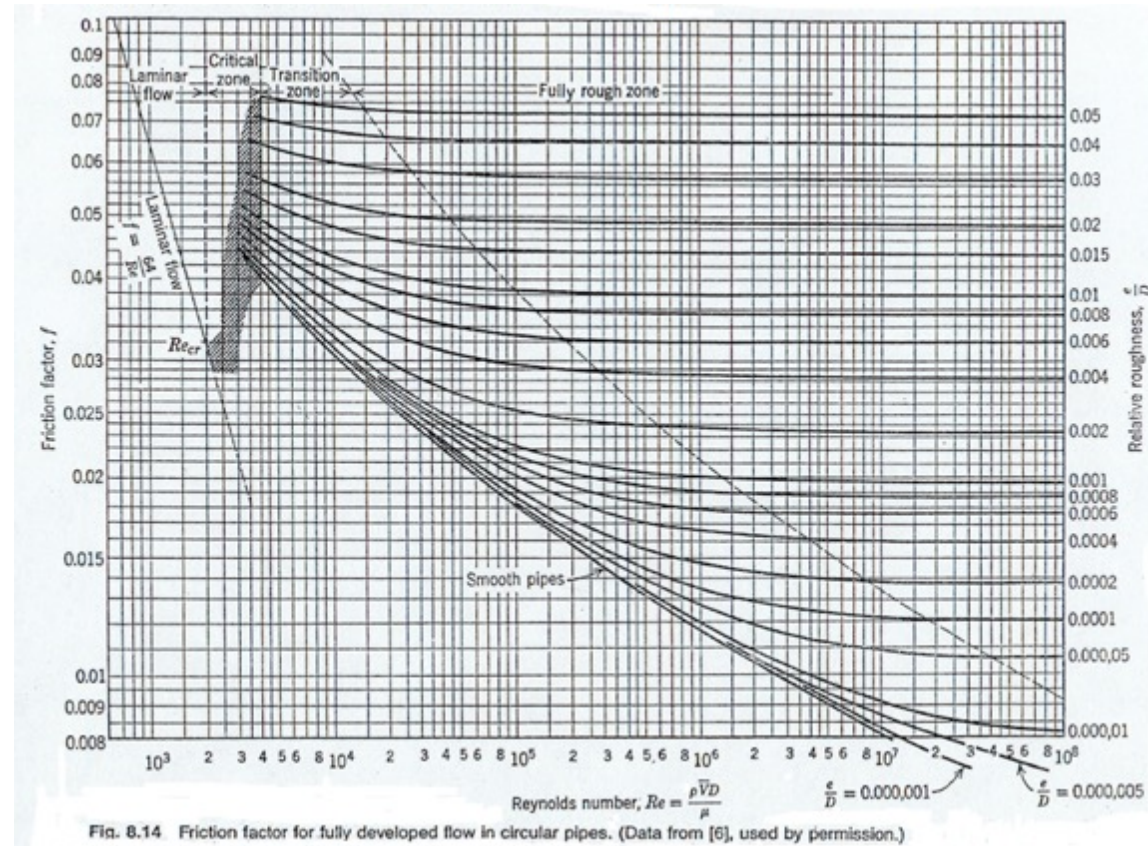
Relaxation in Laminar Flow, $T=0.2$



Relaxation in Laminar Flow, $T=0.3$



What is the Flow Regime in Premise Plumbing?



Legend	Regime	Re Range	1-D Dispersive Transport
	turbulent	$20,000 < Re$	low (probably not important)
	transition	$4,000 < Re < 20,000$	low to moderate (might be important)
	critical	$2,000 < Re < 4,000$	moderate to high (likely to be important)
	laminar	$0 < Re < 2,000$	high (important)

RED and ORANGE conditions likely see dispersive transport in premise plumbing systems

Temp = 50 F / 10 C		Flow Rate (gpm)										
Nominal Diameter (in)	Actual Diameter (in)	0.25	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.250	0.315	1921	3842	7684	11526	15368	19211	23053	26895	30737	34579	38421
0.375	0.430	1407	2815	5629	8444	11258	14073	16887	19702	22517	25331	28146
0.500	0.545	1110	2221	4441	6662	8883	11103	13324	15545	17765	19986	22207
0.750	0.785	771	1542	3083	4625	6167	7709	9250	10792	12334	13876	15417
1.000	1.025	590	1181	2361	3542	4723	5904	7084	8265	9446	10627	11807
1.250	1.265	478	957	1913	2870	3827	4784	5740	6697	7654	8611	9567
Temp = 70 F / 21 C		Flow (gpm)										
Nominal Diameter (in)	Actual Diameter (in)	0.25	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.250	0.315	2572	5144	10289	15433	20578	25722	30866	36011	41155	46300	51444
0.375	0.430	1884	3769	7537	11306	15074	18843	22611	26380	30149	33917	37686
0.500	0.545	1487	2973	5947	8920	11894	14867	17840	20814	23787	26760	29734
0.750	0.785	1032	2064	4129	6193	8257	10322	12386	14450	16515	18579	20643
1.000	1.025	790	1581	3162	4743	6324	7905	9486	11067	12648	14229	15810
1.250	1.265	641	1281	2562	3843	5124	6405	7686	8967	10248	11529	12810
Temp = 120 F / 49 C		Flow (gpm)										
Nominal Diameter (in)	Actual Diameter (in)	0.25	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.250	0.315	4447	8894	17788	26682	35576	44471	53365	62259	71153	80047	88941
0.375	0.430	3258	6515	13031	19546	26062	32577	39093	45608	52124	58639	65155
0.500	0.545	2570	5141	10281	15422	20563	25703	30844	35984	41125	46266	51406
0.750	0.785	1784	3569	7138	10707	14276	17845	21414	24983	28552	32121	35690
1.000	1.025	1367	2733	5467	8200	10933	13667	16400	19133	21867	24600	27333
1.250	1.265	1107	2215	4429	6644	8859	11074	13288	15503	17718	19933	22147
Temp = 140 F / 60 C		Flow (gpm)										
Nominal Diameter (in)	Actual Diameter (in)	0.25	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.250	0.315	5295	10590	21181	31771	42361	52952	63542	74132	84722	95313	105903
0.375	0.430	3879	7758	15516	23274	31032	38790	46548	54306	62064	69822	77580
0.500	0.545	3061	6121	12242	18363	24484	30605	36726	42847	48968	55089	61210
0.750	0.785	2125	4250	8499	12749	16998	21248	25498	29747	33997	38247	42496
1.000	1.025	1627	3255	6509	9764	13018	16273	19527	22782	26037	29291	32546
1.250	1.265	1319	2637	5274	7911	10548	13186	15823	18460	21097	23734	26371

Legend	Regime
	turbulent
	transition
	critical
	laminar

Is Dispersion Important in Premise Plumbing Modeling?

$$\frac{\partial c}{\partial t} = \textcolor{red}{D} \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x} - Kc$$

Depends on product of 2 terms!

**next
lives
here**

Acknowledgments

- National Science Foundation
- American Water Works Assoc Research Foundation
- US Environmental Protection Agency
- Greater Cincinnati Water Works
- Trent Schade
- Greg Wells
- Matt Cutter
- Yeongho Lee
- Zhiwei Li

**next
lives
here**

Questions?

Steven.Buchberger@uc.edu

University of
CINCINNATI



References for Dispersion in Laminar Flow

Cutter, M.R. (2004), "Dispersion in Steady Pipe Flow with Reynolds Number Under 10,000," M.S. thesis, University of Cincinnati, Aug. 2004.

Fischer, H.B., J. List, C. Koh, J. Imberger, N. Brooks (1979) **Mixing in Inland and Coastal Waters**, Academic Press, 302 pages.

Gill, W.N., and R. Sankarasubramanian (1970) "Exact analysis of unsteady convective diffusion", *Proc of the Royal Society of London Series A*, **316**, 341–350.

Y. Lee and S.G. Buchberger (2000) "Is dispersion important in distribution systems?" Proc Joint Conf on Water Resources Engineering, Planning and Management, Minneapolis, MN.

Lee, Y.-H. (2004) "Mass dispersion in intermittent laminar flow," Ph.D. dissertation, University of Cincinnati, May 2004.

Taylor, G.I. (1953) "Dispersion of Soluble Matter in Solvent Flowing Slowly through a Tube," *Proceedings of the Royal Society of London, Series A*, **219**, 186–203.

Taylor, G.I. (1954), "The Dispersion of Matter in Turbulent Flow through a Pipe," *Proceedings of the Royal Society of London, Series A*, **223**, 446–468.

Tzatchkov, V., Aldama, Arreguin (2002) "Advection Dispersion Reaction Modeling in Water Distribution systems" ASCE JWRPM,