-Mixing Processes in Pipes, Sewers & the Natural Environment from Theory to Practice -

The Spatial Routing Procedure for Estimation of Dispersion Coefficient

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CONTENTS

Ⅰ. Backgrounds

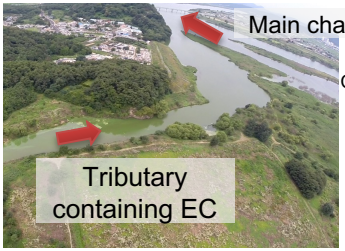
I. Research Background

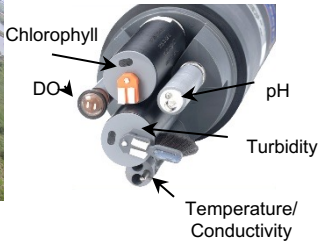
- Concentration Measurements for Estimating Dispersion Coefficients

Optic sensors (Instantaneous source: C-t)

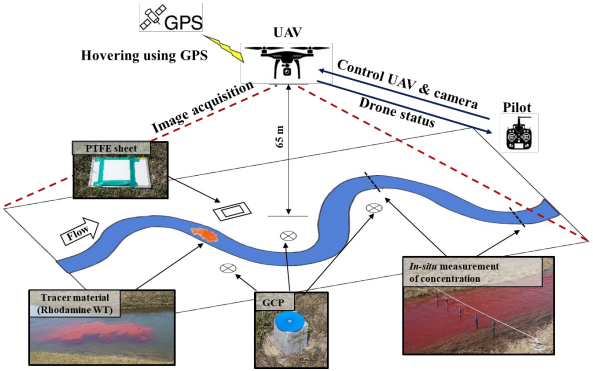
YSI-600OMS

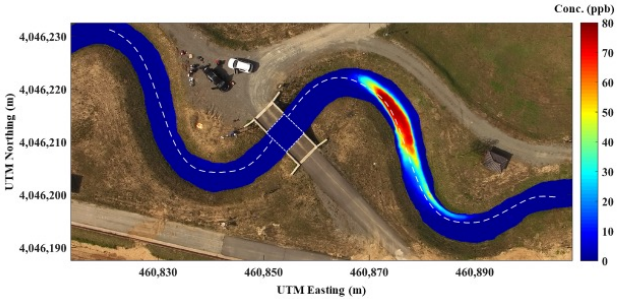
Conductivity meter (Continuous source: C-y)





Aerial Photo (Instantaneous source: Spatial distribution)





Source: Baek (2020)

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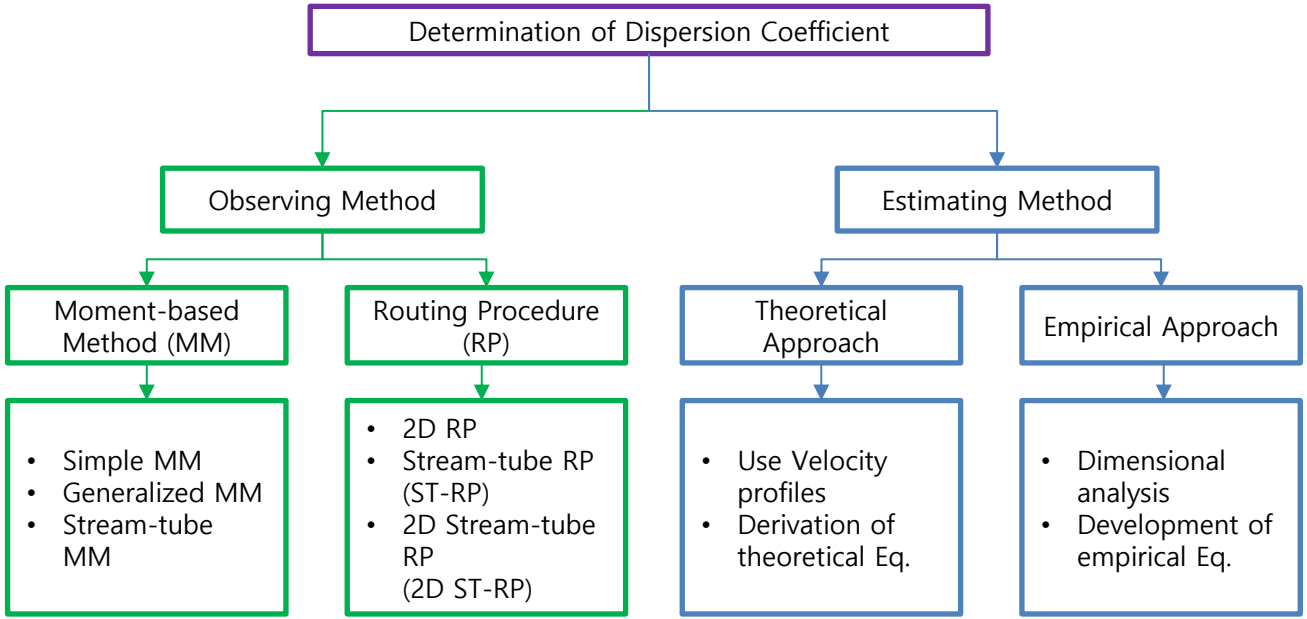
I. Research Background

- Hierarchy of Dispersion Coefficients Estimation



Source: Baek and Seo (2013)

I. Research Background

- Hierarchy of Dispersion Coefficients Estimation

- Moment method (Beltaos, 1980)

$$D_T = \frac{Q^2}{2\psi UH^2}\frac{d\sigma_\eta^2}{dx}\frac{1}{\left[1-\left(1-\eta_0 S_1 - S_0\right)\right]}$$

- 2D Stream Tube Routing Procedure (2D STRP) (Baek and Seo, 2010)

$$C\left(x_2,\eta,t\right) = \int_0^1\int_{-\infty}^{\infty}\frac{C\left(x_1,\omega,\tau\right)}{4\pi\Delta t\sqrt{D_L S_T}}\exp\left(-\frac{U^2\left(t_2-t_1-t+\tau\right)^2}{4D_L\Delta t}\right)\exp\left(-\frac{\left(\eta-\omega\right)^2}{4S_T\Delta t}\right)d\tau d\omega$$

- Velocity-based method (Dispersion tensor) (Fischer et al., 1979)

$$D_{ij} = -\frac{1}{h}\int_0^h u_i'\int_0^z\frac{1}{\varepsilon_v}\int_0^z u_j'\,dzdzdz$$

I. Research Background

- Derivation of 2D ST-RP (Baek and Seo, 2010)

- Analytic solution of 2D Advection-Dispersion equation in a stream-tube coordinates

$$C(x, y, t_2) = \int_0^1 \int_{-\infty}^{\infty} \frac{C(\zeta, \omega, t_1)}{4\pi\Delta t\sqrt{D_L D_T}} \exp\left(-\frac{(x-\zeta)^2}{4D_L\Delta t}\right) \exp\left(-\frac{(y-\omega)^2}{4D_T\Delta t}\right) d\zeta\, d\omega$$

$C(x, y, t_1)$: upstream concentration distribution

$C(x, y, t_2)$: downstream concentration distribution

1) Frozen-cloud approximation: spatial profile → temporal concentration profile
2) Stream-tube concept

$$C(x_2, \eta, t) = \int_0^1 \int_{-\infty}^{\infty} \frac{C(x_1, \omega, \tau)}{4\pi\Delta t\sqrt{D_L S_T}} \exp\left(-\frac{U^2(t_2 - t_1 - t + \tau)^2}{4D_L\Delta t}\right) \exp\left(-\frac{(\eta-\omega)^2}{4S_T\Delta t}\right) d\tau\, d\omega$$

$\eta = q/Q$: Dimensionless discharge

$S_T = UB_c = \frac{\psi H^2 U^2}{Q^2} D_T$ $\quad \psi = \frac{1}{Q}\int_0^Q h^2 u dq$: Dimensionless shape factor

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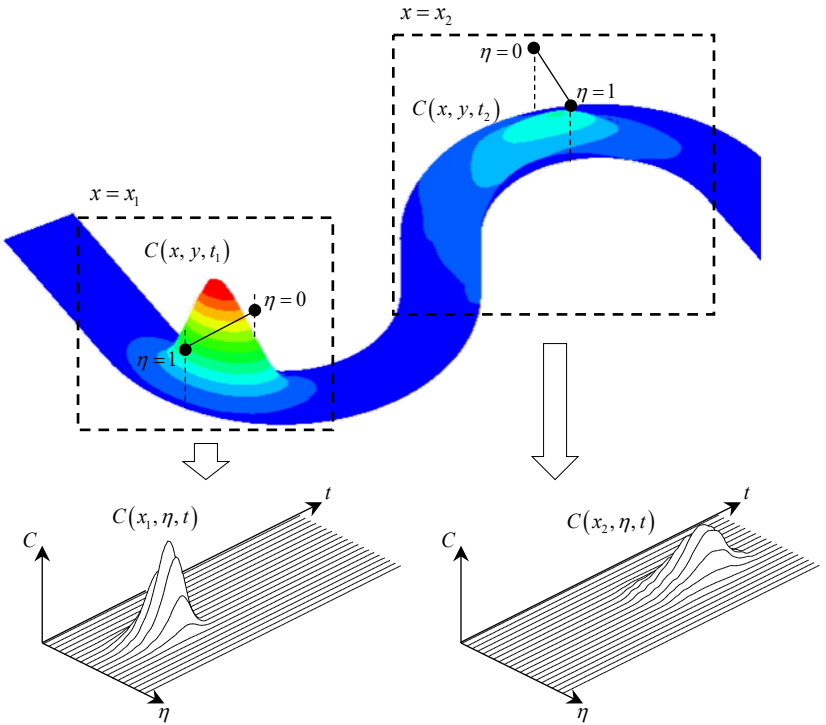
I. Research Background

- Comparisons of 2D S-RP and 2D STRP



- Use spatial distribution: 2D S-RP

$$C_{pred}(x, y, t_2) = \int_x \int_y C(x, y, t_1) f(D_L, D_T)$$

Find best fit ⇨ Find D_L, D_T

$$C(x, y, t_2)$$

- Use temporal concentration: 2D STRP

$$C_{pred}(x_2, \eta, t) = \int_y \int_t C(x_1, \eta, t) f(D_L, B_C)\, dt\, dy$$

Find best fit ⇨ Find D_L, D_T

$$C(x_2, \eta, t)$$

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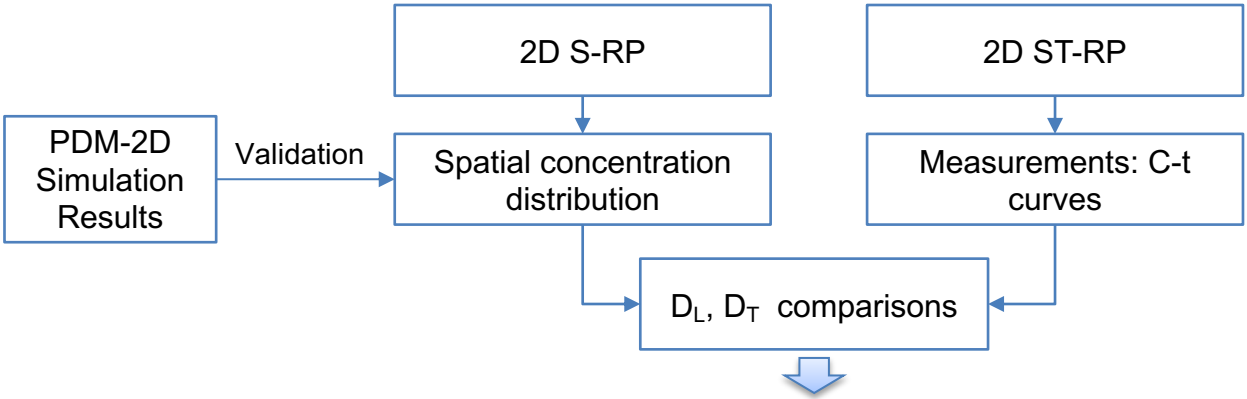
I. Research Background

- Research Purposes

  - Skewed concentration curves due to channel irregularities and storage effects



Case 1) Frozen cloud assumption for 2D STRP

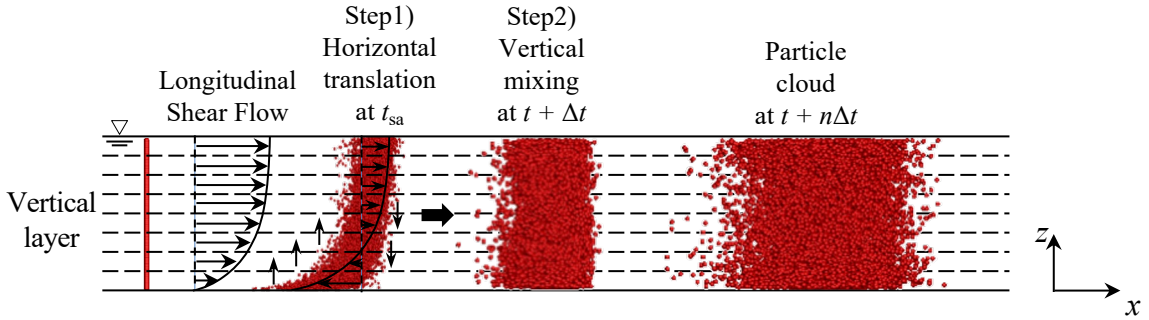
Case 2) Applicability of routing procedures for skewed concentration curves

Ⅱ. Methods

Ⅱ. Methods

- Outlines of PDM-2D (Particle Dispersion Model-2D)

  - Sequential Computation of two steps of shear dispersion: horizontal translation and vertical diffusion Using Particle Tracking Technique (Park and Seo, 2018)
  - Dispersion coefficients are not necessary



Step1) Horizontal translation

$$\tilde{x}_i^k\left(z_{sa},\, t_{sa}\right) = x_i^k\left(z_{sa},\, t\right) + u_i\left(x_i^k,\, z_{sa},\, t\right)\Delta t + R\sqrt{2\varepsilon_h \Delta t}$$

Step2) Vertical diffusion

Full-column mixing

$$x_i^k\left(z,\, t+\Delta t\right) = \tilde{x}_i^k\left(a\frac{h}{L},\, t_{sa}\right)$$

Random mixing

$$x_i^k\left(z,\, t+\Delta t\right) = \tilde{x}_i^k\left(z_{sa} + R\sqrt{2\varepsilon_z \Delta t},\, t_{sa}\right)$$

Ⅱ. Methods

- Test Cases

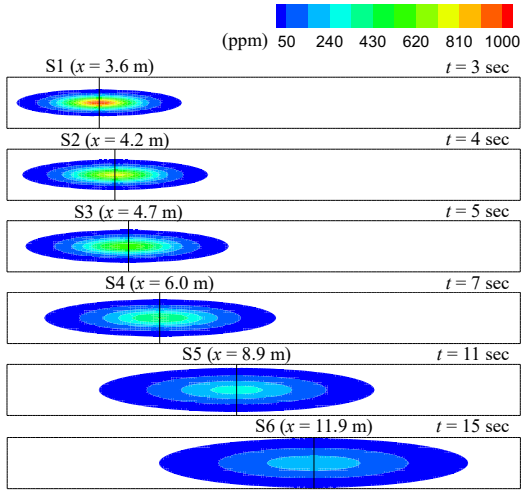
Case 1) Unsteady flow in straight channel

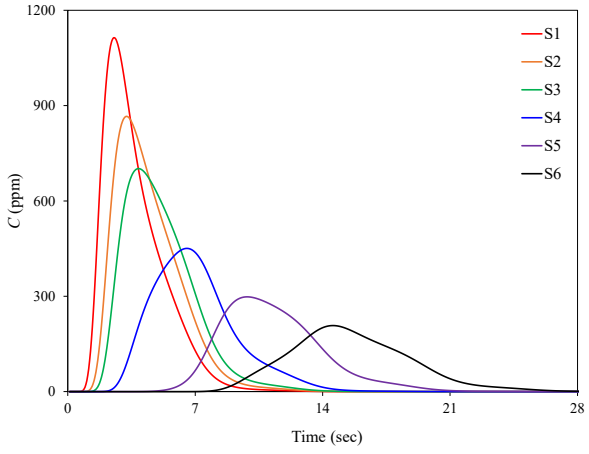
$$C(x,y,t)=\frac{M}{4\pi t\sqrt{D_L D_T}}\exp\left[-\frac{\left(x-x_0-u(t)\tau\right)^2}{4D_L t}-\frac{\left(y-y_0\right)^2}{4D_T t}\right]$$

$$D_L = 5.93hu^* = 0.163\ \mathrm{m^2/s}$$

$$D_T = 0.15hu^* = 0.0041\ \mathrm{m^2/s}$$

$$u(t)=0.25\sin t+0.7$$



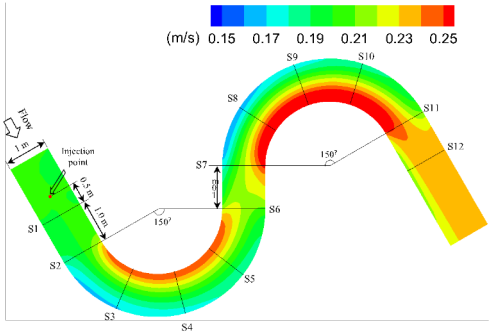


Ⅱ. Methods

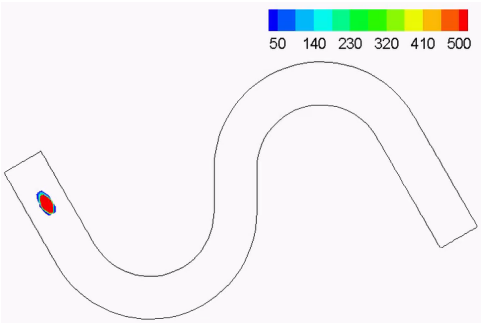
- Test Cases

Case 2) Meandering channel

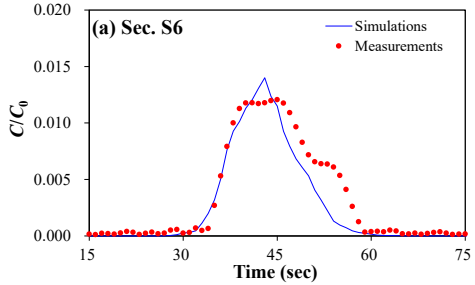
Velocity magnitude (Q = 0.06 m³/s, H=0.4 m)

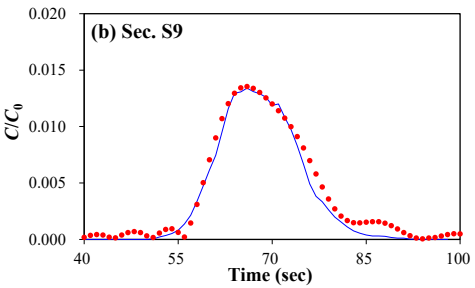


Results of PDM-2D (No. of particles = 30,000)



Comparisons of C-t curves





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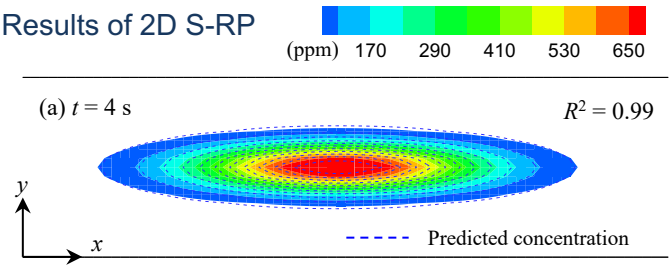
Ⅲ. Results

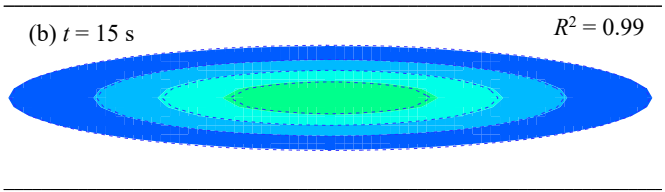
Ⅲ. Results

- Case 1) Unsteady flow in straight channel

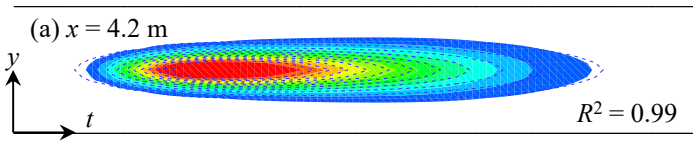
  - Spatial concentration distribution: Gaussian distribution
  - C-t curves: Skewed concentration curves due to unsteady flow
  - Prediction results in 2D STRP include more errors than 2D S-RP

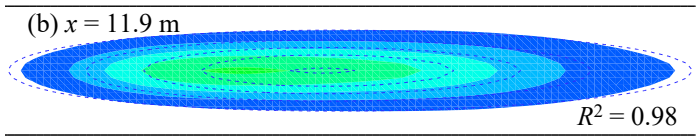
Results of 2D S-RP





Results of 2D STRP





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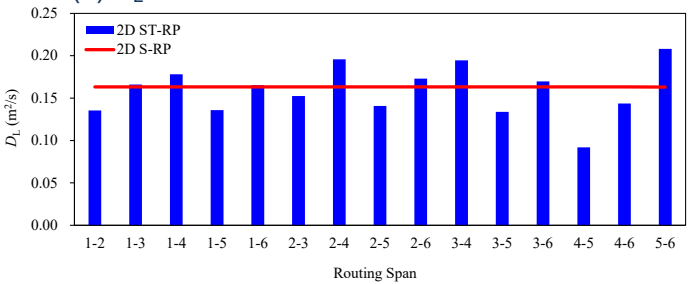
Ⅲ. Results

- Case 1) Unsteady flow in straight channel

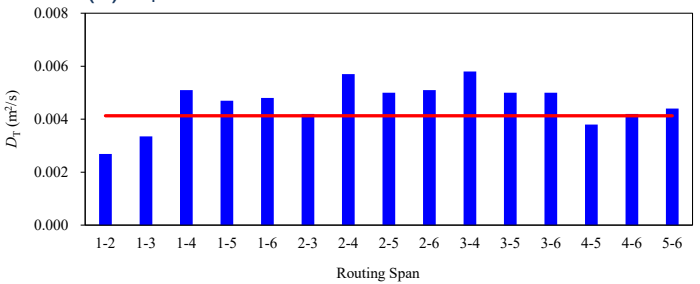
  - Results of 2D STRP depend on selection of the routing span
  - 2D STRP shows errors in the range of 1.2–43.7% for D_L and 1.8–40.6% for D_T
  - Pollutant cloud experiences variations in shear dispersion as it passes through the measurement section. ⇨ Frozen cloud assumption doesn't hold for unsteady flow

Routing Span	D_L (m²/s)			D_T (m²/s)		
	2D STRP	2D S-RP	Input value	2D STRP	2D S-RP	Input value
From S1	0.135	0.163	0.163	0.0048	0.0041	0.0041
From S2	0.166	0.163		0.0047	0.0041	
From S3	0.178	0.163		0.0051	0.0041	
From S4	0.136	0.163		0.0033	0.0041	
From S5	0.165	0.163		0.0027	0.0041	
Avg.	0.159	0.163		0.0046	0.0041	

(a) D_L



(b) D_T

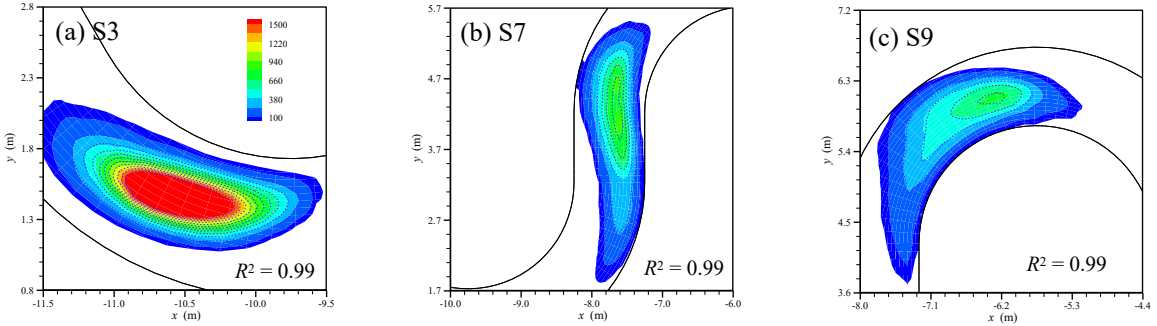


Ⅲ. Results

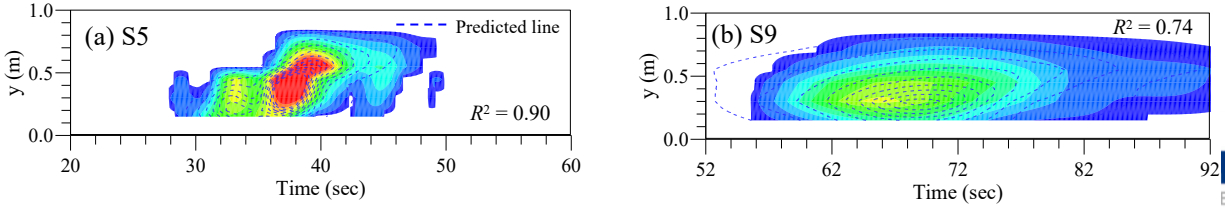
- Case 2) Meandering Channel

- Both C-t curves and spatial distributions have skewed distributions ⇨ Non-Fickian mixing
- 2D STRP shows relatively inaccurate results compared to the results obtained by 2D S-RP

Prediction Results of 2D S-RP



Prediction Results of 2D STRP



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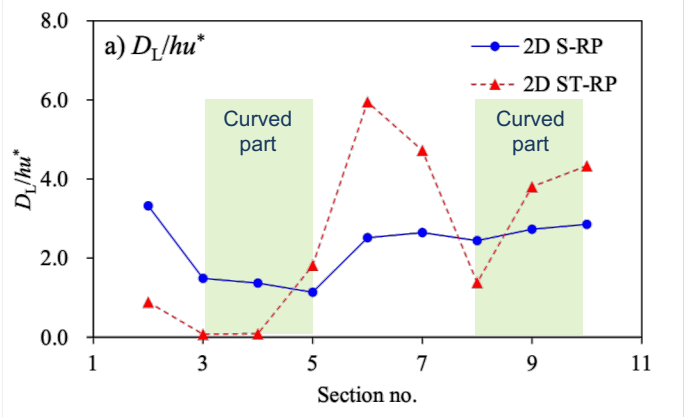
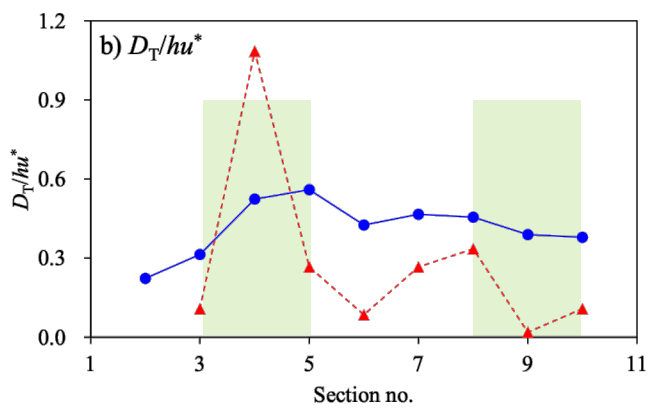
Ⅲ. Results

- Case 2) Meandering Channel

- Overall variability along the channel of the results by 2D STRP are greater than those of the 2D S-RP





Compare the results with the velocity based method proposed by Fischer et al. (1979)

Velocity measurements $D_{ij} = -\frac{1}{h}\int_0^h u_i' \int_0^z \frac{1}{\varepsilon_v} \int_0^z u_j' \, dzdzdz$

D_L and D_T are converted to tensor format by the coordinate transform

$$D_{xx} = D_L \frac{\overline{u}^2}{\overline{u}^2+\overline{v}^2} + D_T \frac{\overline{v}^2}{\overline{u}^2+\overline{v}^2} \qquad D_{xy} = D_{yx} = (D_L - D_T)\frac{\overline{u}\,\overline{v}}{\overline{u}^2+\overline{v}^2} \qquad D_{yy} = D_T \frac{\overline{u}^2}{\overline{u}^2+\overline{v}^2} + D_L \frac{\overline{v}^2}{\overline{u}^2+\overline{v}^2}$$

Ⅲ. Results

- Case 2) Meandering Channel

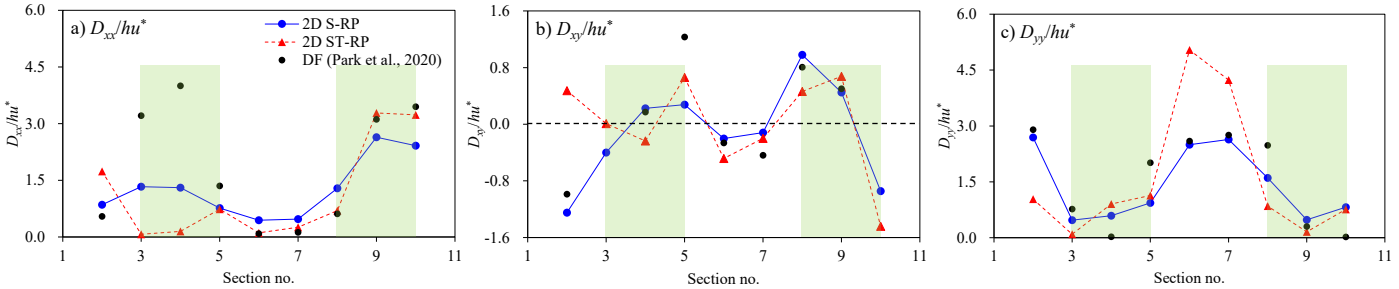
- Relative discrepancies in comparisons to the velocity-based method (DF)

Section	D_{xx}	D_{xy}=D_{xy}	D_{yy}
1st bend			
Cross-over			
2nd bend			

: 2D STRP

: 2D S-RP

The results obtained by 2D S-RP are more similar to the velocity-based method than those obtained by 2D STRP



IV. Conclusions & Future Studies

Ⅳ. Conclusions

- Validity of the Frozen Cloud Assumption

  - Not necessary when a pollutant cloud passes through a probe fast enough (Chatwin, 1971)

    ⇨ For unsteady flow (increasing the retention time), tracer experiences dispersion while it crosses a cross-section, and C-t curves make skewed concentration curves

    ⇨ 2D STRP exhibits errors depending on the selections of routing span for the skewed concentration curves

- Applicability to Non-Fickian Mixing

  - Tracer cloud in the meandering channel exhibits a skewed distribution in both the spatial and temporal distributions

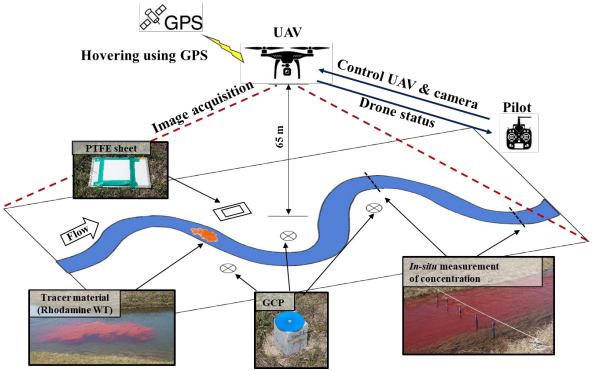
    ⇨ Both the results obtained by the 2D STRP and the 2D S-RP show discrepancies compared to the velocity-based method

    ⇨ The results obtained by the 2D S-RP are more comparable to the velocity-based method than those obtained by the 2D STRP

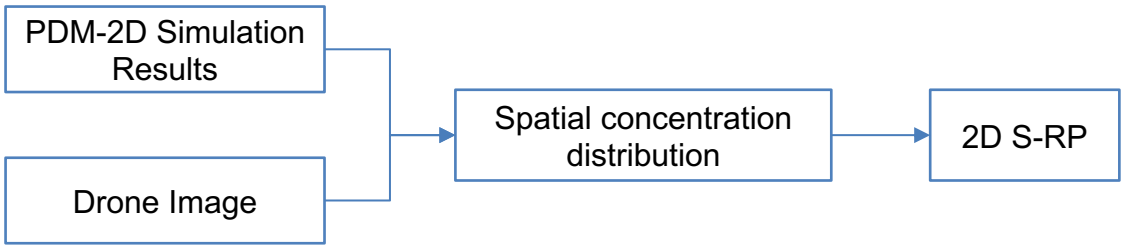
    ⇨ Inherent limitations of 2D STRP and 2D S-RP due to the Fickian dispersion

Ⅳ. Future Studies

- Spatial distribution obtained by a drone Image



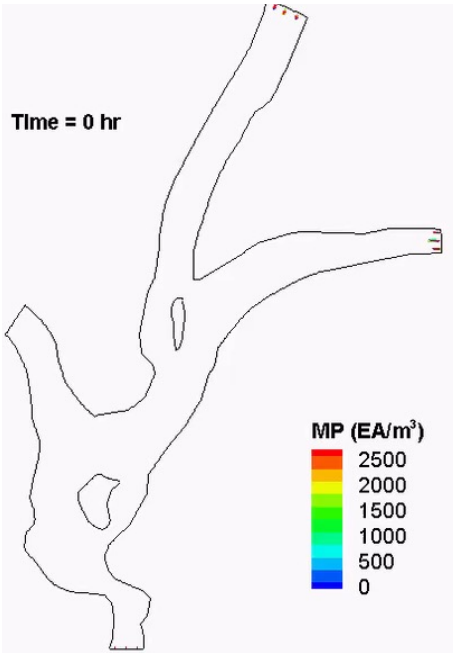
(source: Baek et al., 2019)

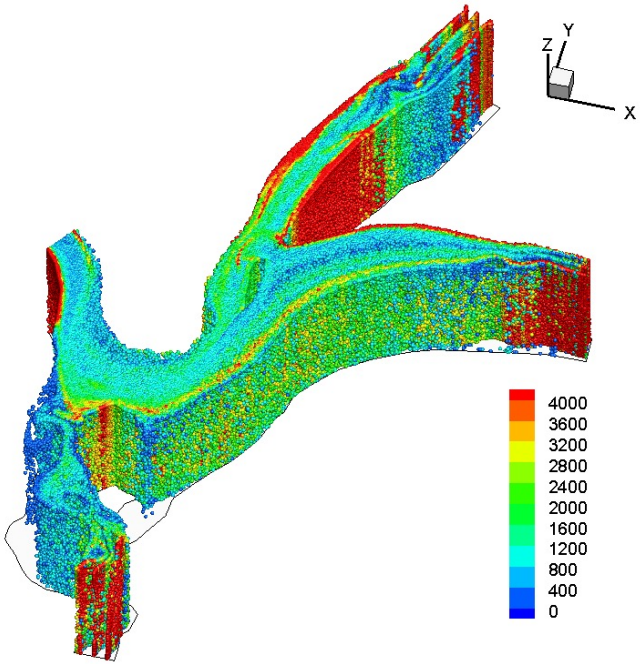


IV. Future Studies

Application of PDM-2D

MPT-Q3D (Microplastic Transport Modeling in River Environment)





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Thank You