Advection-Diffusion Solver Suitable for Fluid Circulation in Drilling

Johnny Petersen Ph.D.

formerly NORCE*, Norway

(formerly IRIS, Norway)

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Mixing Processes in Pipes, Sewers & the natural Environment from Theory to Practice

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*NORCE Norwegian Research Centre

Overview

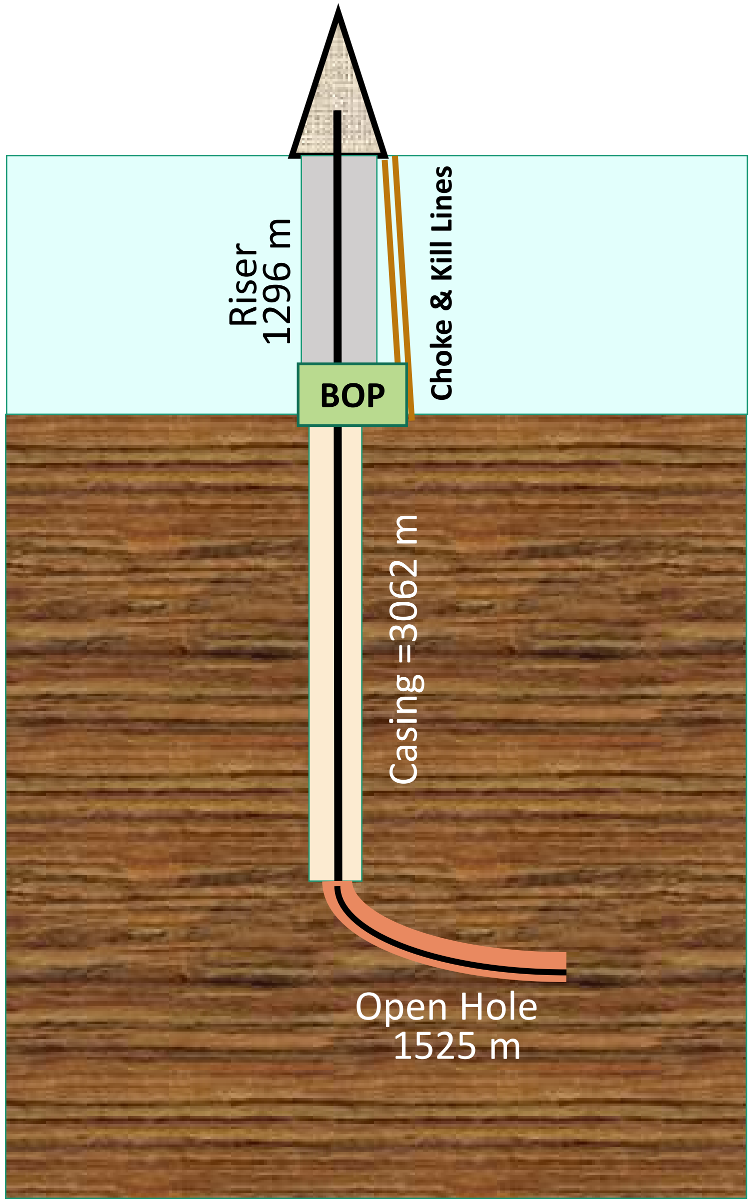
- **Computational Challenges in Off-Shore Drilling**
- **1-D Advection**
- **Numerical diffusion reduction method**
- **Advection-*Diffusion-Operator* Algorithm**
- **Standard Adv-Diff. Method, FTCS**
- **Testing Advection-Diffusion Algorithm**

- **Laminar Dispersion**
- **Ideas about Transition Dispersion**

- **Conclusions**

Off-Shore Drilling Challenges

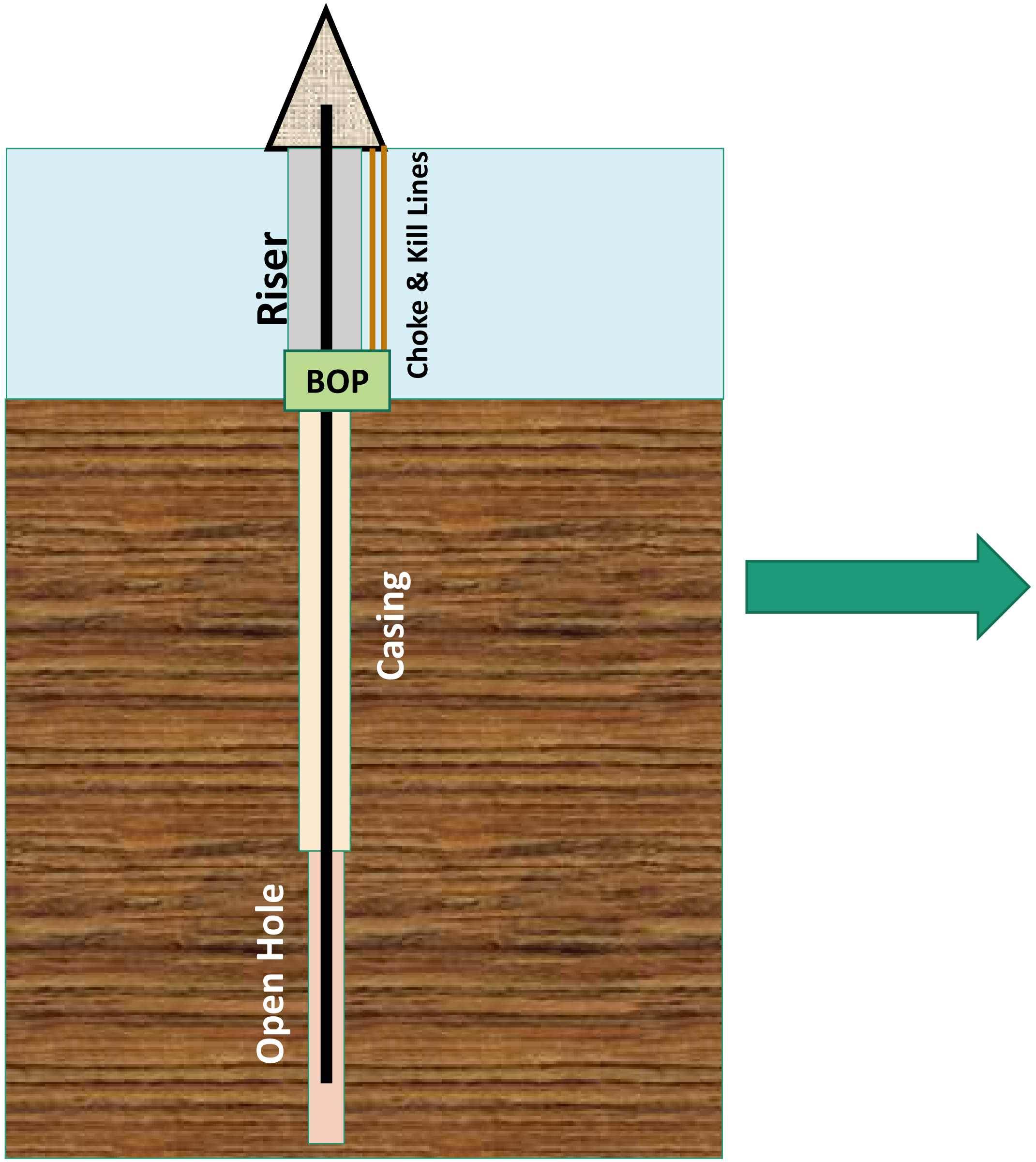
- Depths of several thousand meters
- Pump rates of thousands of liters per minute
- Open Hole and Casing ID ~ 12” – 13”
- Drill String OD ~ 5.5”
- Temperatures range: 0°C – 200°C   (HPHT)
- Pressure range: 1 bar – 1400 bar   (HPHT)
- Hydrocarbon and Drilling Mud Chemistry
  - Volume and phase changes (2 phase flow)
  - Interaction between Oil Based Mud and Gas/Oil influx from well
- Using mud “pills” (Well cleaning, MPD operations, Cementing, etc..)



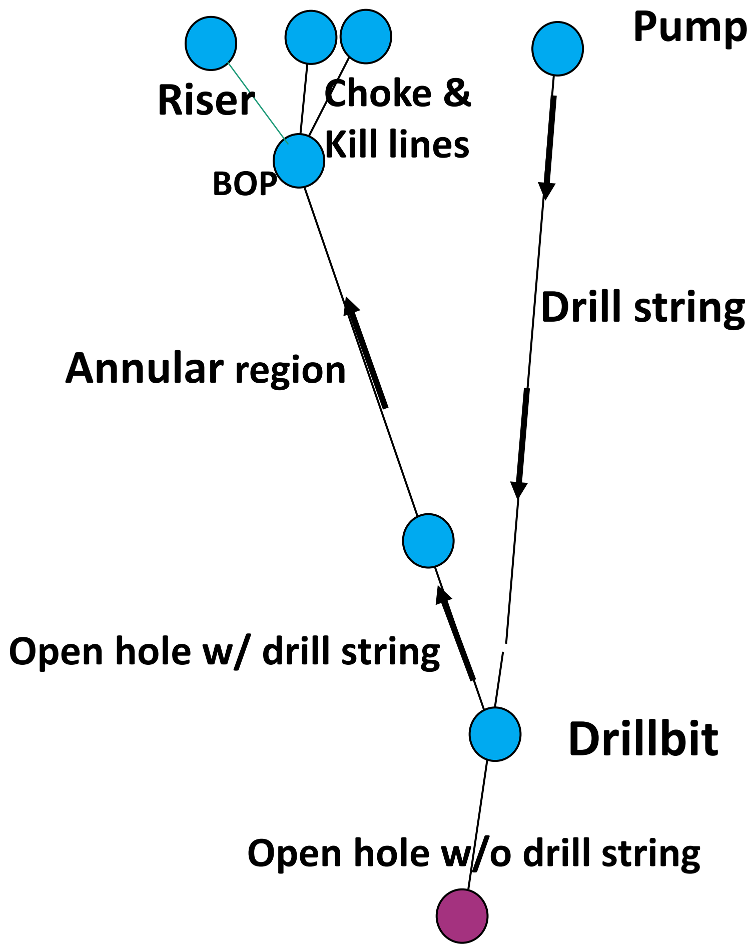
Example well (not HPHT)

Dynamic simulations can not be slower than real time!!

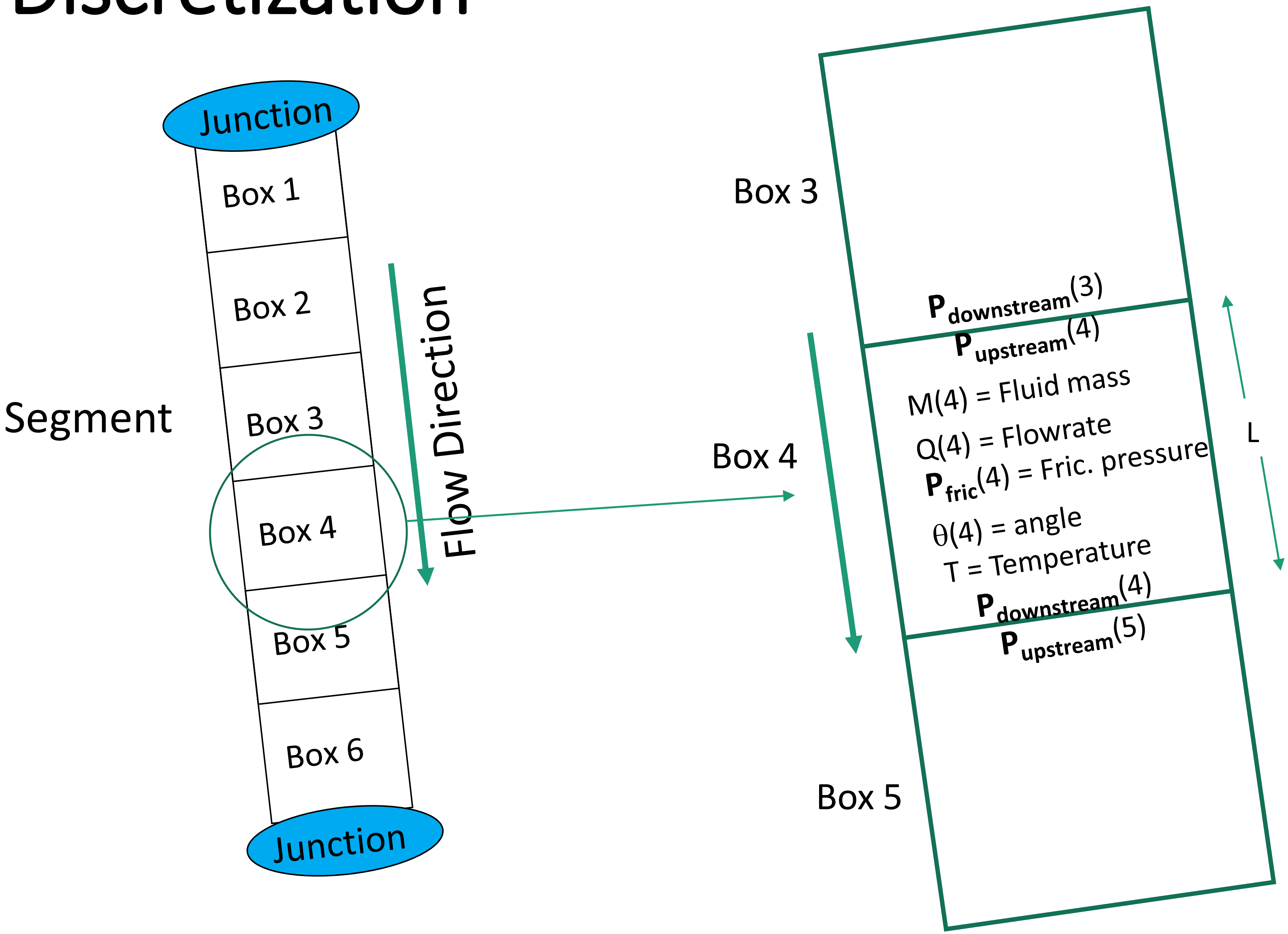
Divide and Conquer Approach



› Example (7 segments)



Discretization



Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

Integrate over a pipe box volume:

$$\iiint_{Vol(i)} \frac{\partial \rho}{\partial t} \; dVol + \iiint_{Vol(i)} \nabla \cdot (\rho \boldsymbol{u}) \, dVol = 0$$

$$\frac{\iiint_{Vol(i)} \rho(t + \Delta t) \, dVol - \iiint_{Vol(i)} \rho(t) \, dVol}{\Delta t} + \oiint_{surface(i)} \rho \boldsymbol{u} \cdot \widehat{\boldsymbol{n}} \; dS = 0$$

$$\text{Mass}(t+\Delta t) - \text{Mass}(t) - A_u \, \rho_u \, U_{s,u} \, \Delta t + A_d \, \rho_d \, U_{s,d} \, \Delta t = 0$$

$$\text{Mass}(i, t+\Delta t) = \text{Mass}(i, t) + \Delta\text{Mass}_u(i, t) - \Delta\text{Mass}_d(i, t)$$

Conservation of Momentum

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \rho \mathbf{g} - \nabla P - \nabla \cdot \overleftrightarrow{\boldsymbol{\tau}}$$

Acceleration term

Bernoulli effect

Shear stress tensor

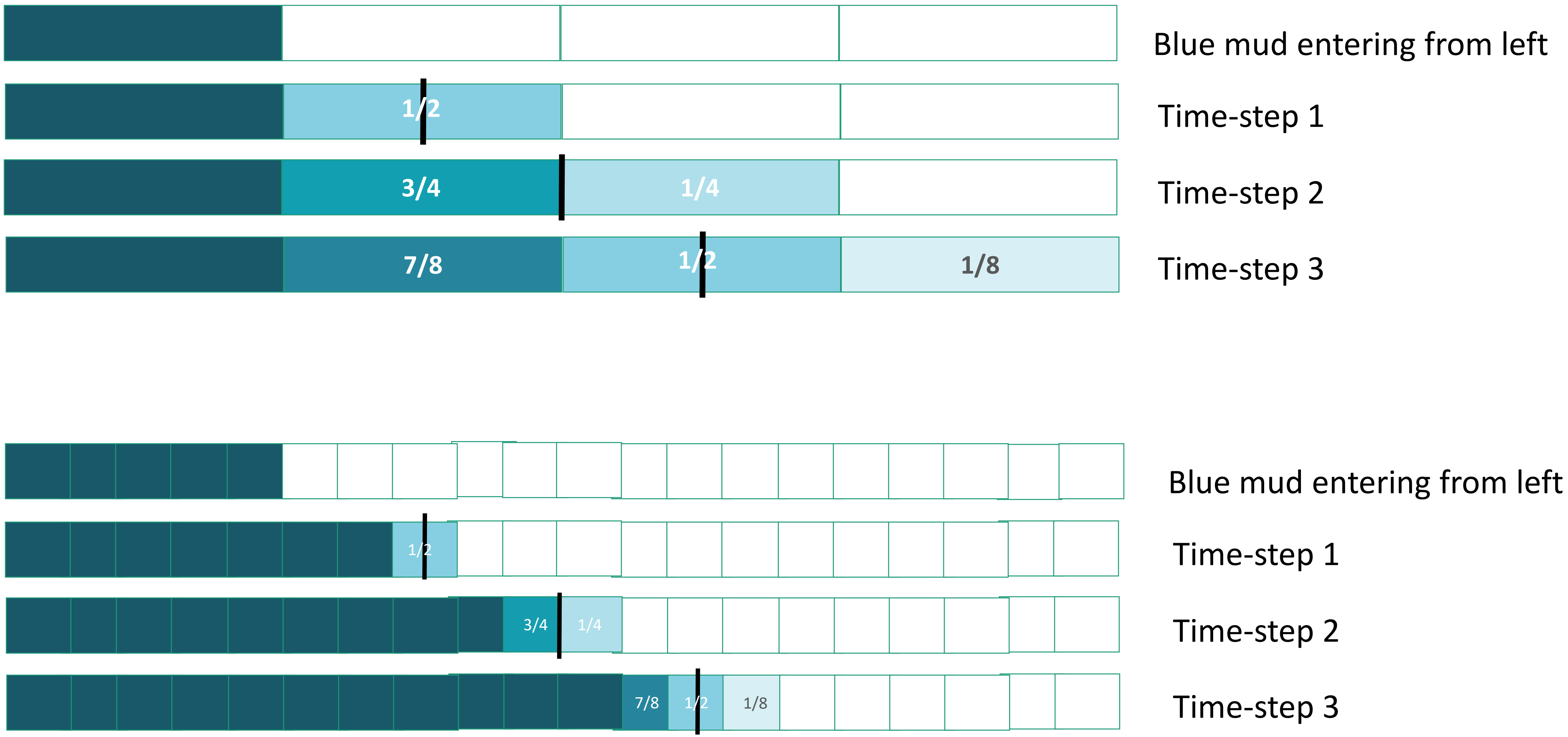
- Set Acceleration term = 0
  - Assume steady state circulation
  - Ignore sound pulses

- Ignore Bernoulli effect
  - Small effect
  - Non-permanent

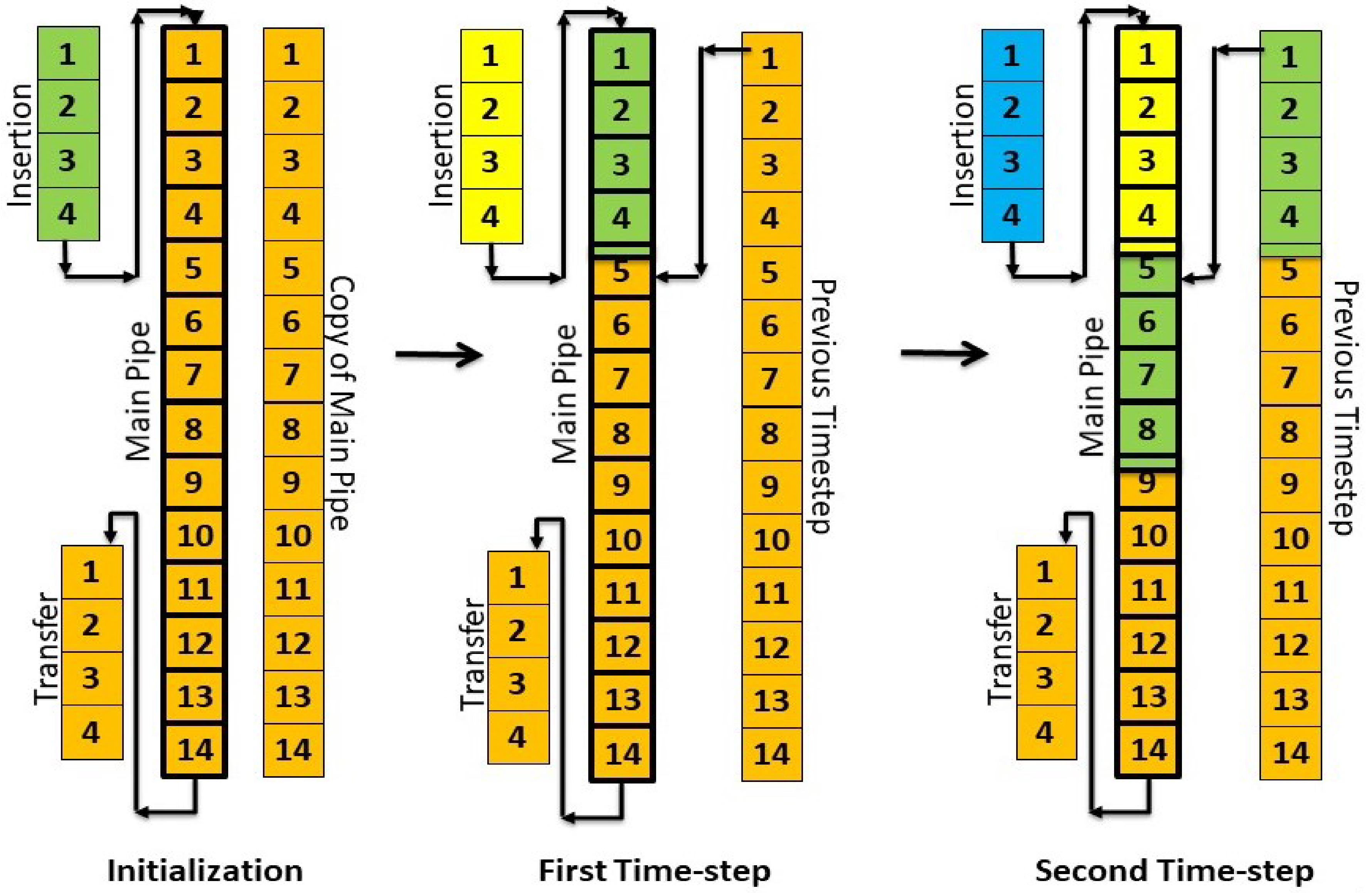
Divergence of Shear Stress Tensor: Frictional pressure gradient due to fluid viscosity and pipe roughness

$$P_{down} = P_{up} - P_{fric} - \rho_{average}\, g\, L \cos\theta$$

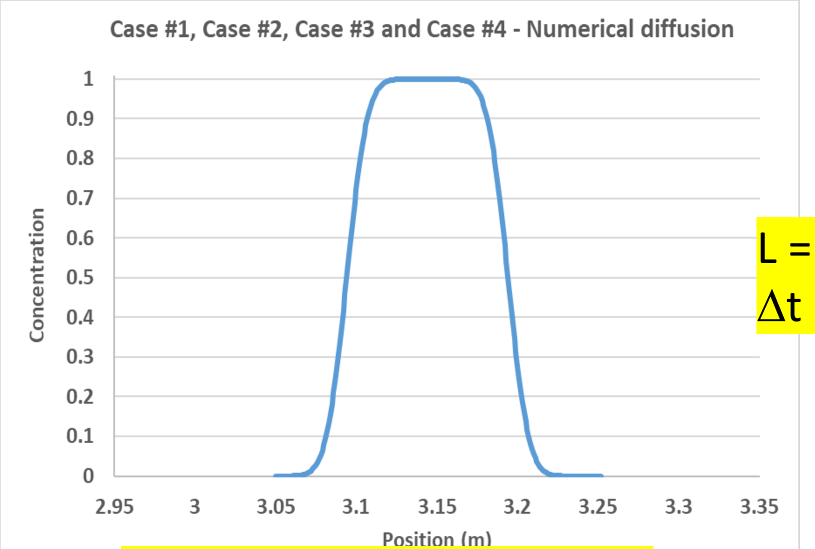
Sub-Boxes reduce numerical diffusion



Circulation Example with Sub-Boxes   N > 4

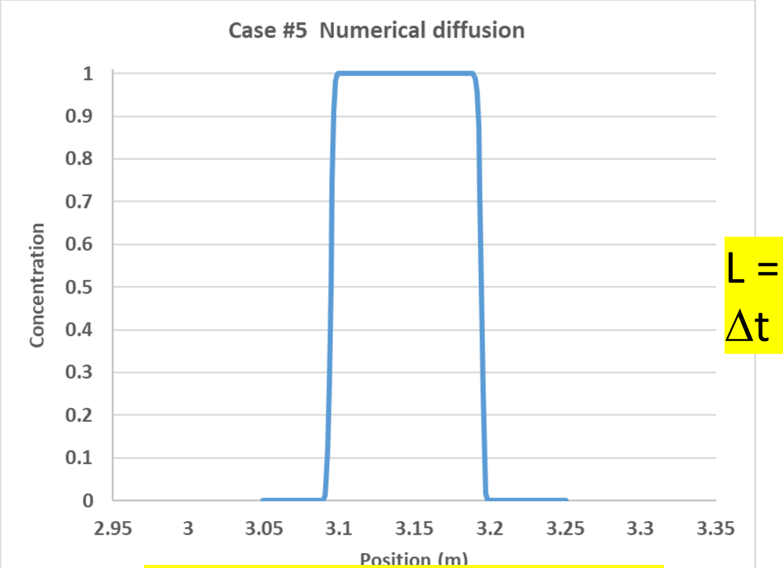


Numerical Diffusion Examples



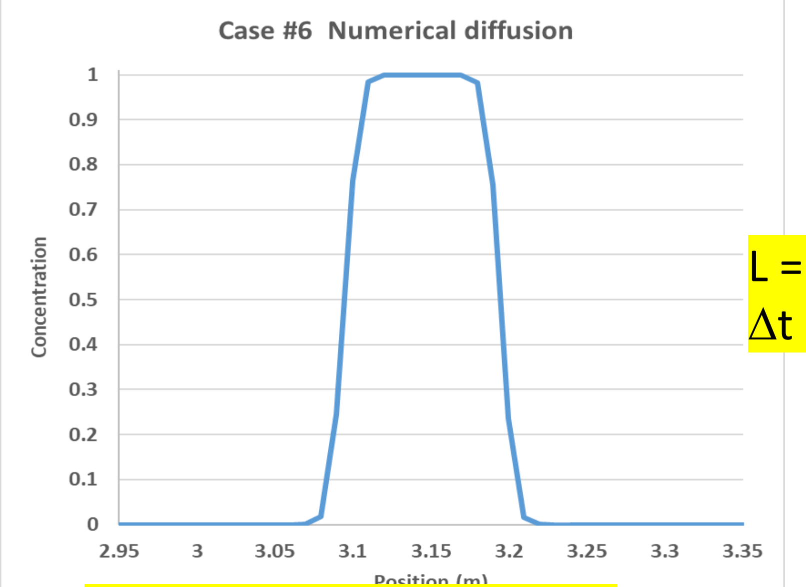
L = 0.1 cm
Δt = 0.01 s

400 timesteps, v Δt/L = 7.5



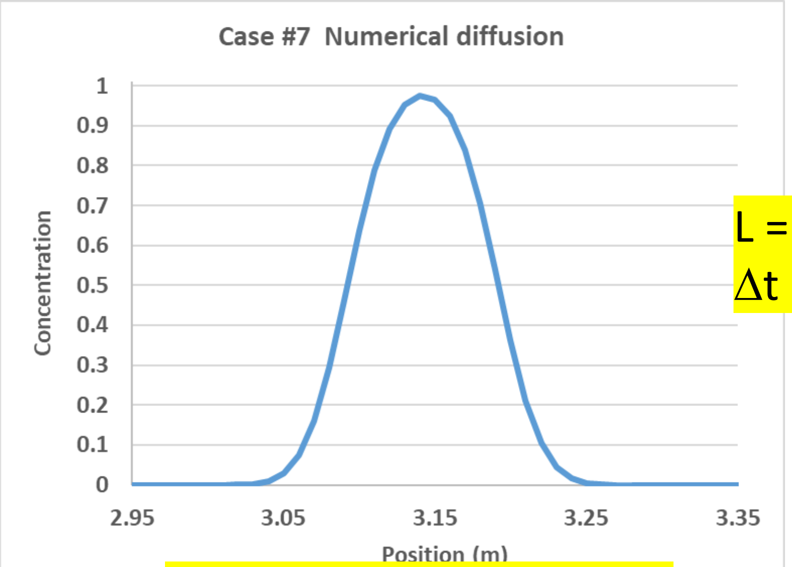
L = 0.05 cm
Δt = 0.01 s

400 timesteps, v Δt/L = 15



L = 0.05 cm
Δt = 0.005 s

800 timesteps, v Δt/L = 7.5



L = 0.5 cm
Δt = 0.05 s

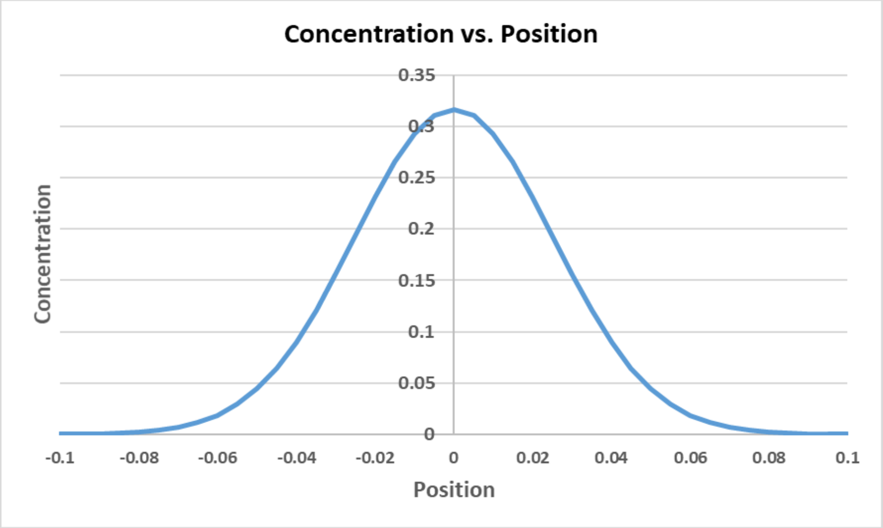
80 timesteps, v Δt/L = 7.5

Pro’s and Con’s

- Pro’s
  - Higher spatial resolution
  - Numerical diffusion decreases rapidly with N
  - No Front-tracking
  - Certain computations become simpler (simpler iteration in computing $P_{downstream}$ in each box)
  - Modelling physical dispersion between fluid fronts is possible
- Con’s
  - Increased number of boxes - some computation increases with the extra number of boxes
  - Book-keeping of masses going through a box in one time-step

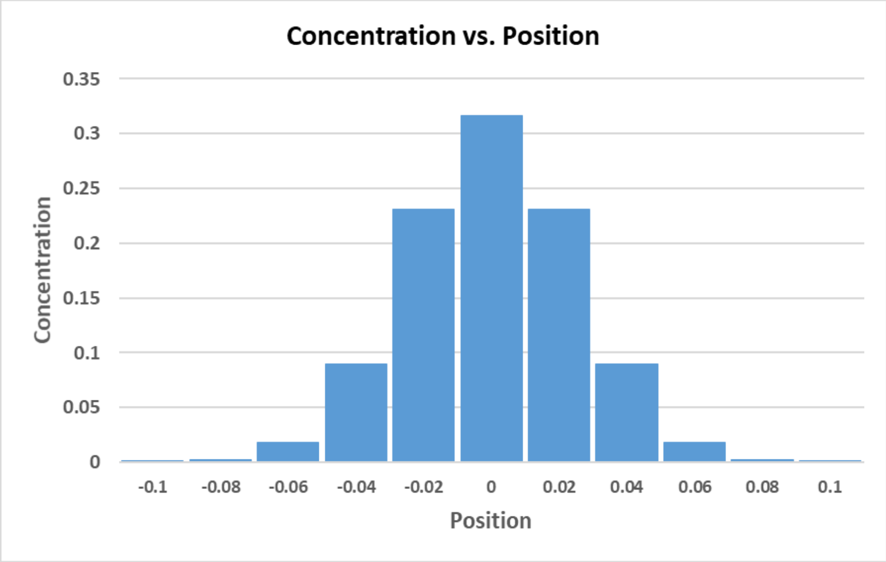
Diffusion Equation

1-d diffusion eq.: $$\frac{\partial C}{\partial t} + u\,\frac{\partial C}{\partial x} = D\,\frac{\partial^2 C}{\partial x^2}$$



Continuous space

- A point source will diffuse with a Gauss distribution
- Width of the Gauss curve is only function of time and diffusion coefficient, D.
- The diffusion curve will spread the same when u=0, as when $u = u_0$ and the observer travels with $u = u_0$.
- Simplify by solving $$\frac{\partial C}{\partial t} = D\,\frac{\partial^2 C}{\partial x^2}$$



Discretized space

Computing diffusion operator

- A pipe stretching from –X to +X, is discretized in many boxes, each with box length, L.
- All boxes are filled with fluid, but the box in the “middle”, x=0, is filled with a tracer (unit volume concentration).
- After a time step, Δt, with diffusion constant, D, the tracer concentration, or *Dispersion Weight*, DW, in the nearby boxes are:

$$\mathrm{DW}(j) = 0.5\ \left[\operatorname{erf}\left(\frac{jL + \frac{L}{2}}{\sqrt{4\pi D\Delta t}}\right) - \operatorname{erf}\left(\frac{jL - \frac{L}{2}}{\sqrt{4\pi D\Delta t}}\right)\right]$$

“erf” is the error function, and j=0 at the “middle” position.

Computing diffusion operator weights

- DW(j) approaches zero as j approaches +/- ∞
- Truncation needed!
- Decide on a truncation value, TruncVal. Example, Trunc Val = 0.99
  - Find ***NDmax*** such that

$$\mathrm{DW}(0) + \sum_{j=1}^{NDmax} 2 \times \mathrm{DW}(j) \geq 0.99$$

- Problem: Does not conserve mass!

Compare 2 Methods/Fixes that will conserve mass:

Simple Method (SM)

$$\mathrm{DW}_{\mathrm{SM}}(0) = 1 - \sum_{j=1}^{NDmax} 2 \times \mathrm{DW}(j)$$

Truncated Gauss(TG)

$$\mathrm{DWSum} = 1 + \sum_{j=1}^{NDmax} 2 \times \mathrm{DW}(j)$$

$$\mathrm{DW}_{\mathrm{TG}}(j) = \mathrm{DW}(j)/\mathrm{DWSum}$$

Diffusion/Turbulent Flow Dispersion Operation

Before Diffusion/After Advection: Mass of type **k** in box **i** is $\mathbf{Mass_0(k,i)}$

After the Diffusion, the new mass of type **k** in box **i**, **Mass(k,i)**, is:

$$\mathbf{Mass(k,i) = Mass_0(k,i) \times DW(0) + \sum_{j=1}^{NDmax} DW(j) \times (Mass_0(k,i\text{-}j) + Mass_0(k,i+j))}$$

The truncation of the Gauss curve effectively reduces the Diffusion Constant.

Introduce Multiplicative Correction Factor to obtain optimal operator for Simple Method and Truncated Gauss, $\mathbf{MCF_{SM}}$ and $\mathbf{MCF_{TG}}$.

Simple Method

$\mathbf{D_{SM} = MCF_{SM} \times D}$

Truncated Gauss

$\mathbf{D_{TG} = MCF_{TG} \times D}$

MCF_{SM} and MCF_{TG} are found through iterative search.

Example

Fluid (salt water)	Salinity 1 g/kg
Density	999 kg/m^3
Temperature	20°C
Kinematic Viscosity	1.0047×10^{-6} m^2/s
Superficial Flow Velocity, U_s	0.75 m/s
Pump Rate	45 l/min
Pipe Diameter, d	0.0356825 m
Reynolds Number, Re	26636
Diffusion Coefficient	0.011243 m^2/s
Tracer Volume	1 dl
Initial tracer position	From 0.1 m to 0.2 m

$$D = U_s\, d\, (1.17 \cdot 10^9\, Re^{-2.5} + 0.41) \qquad 3000 < Re < 50000$$

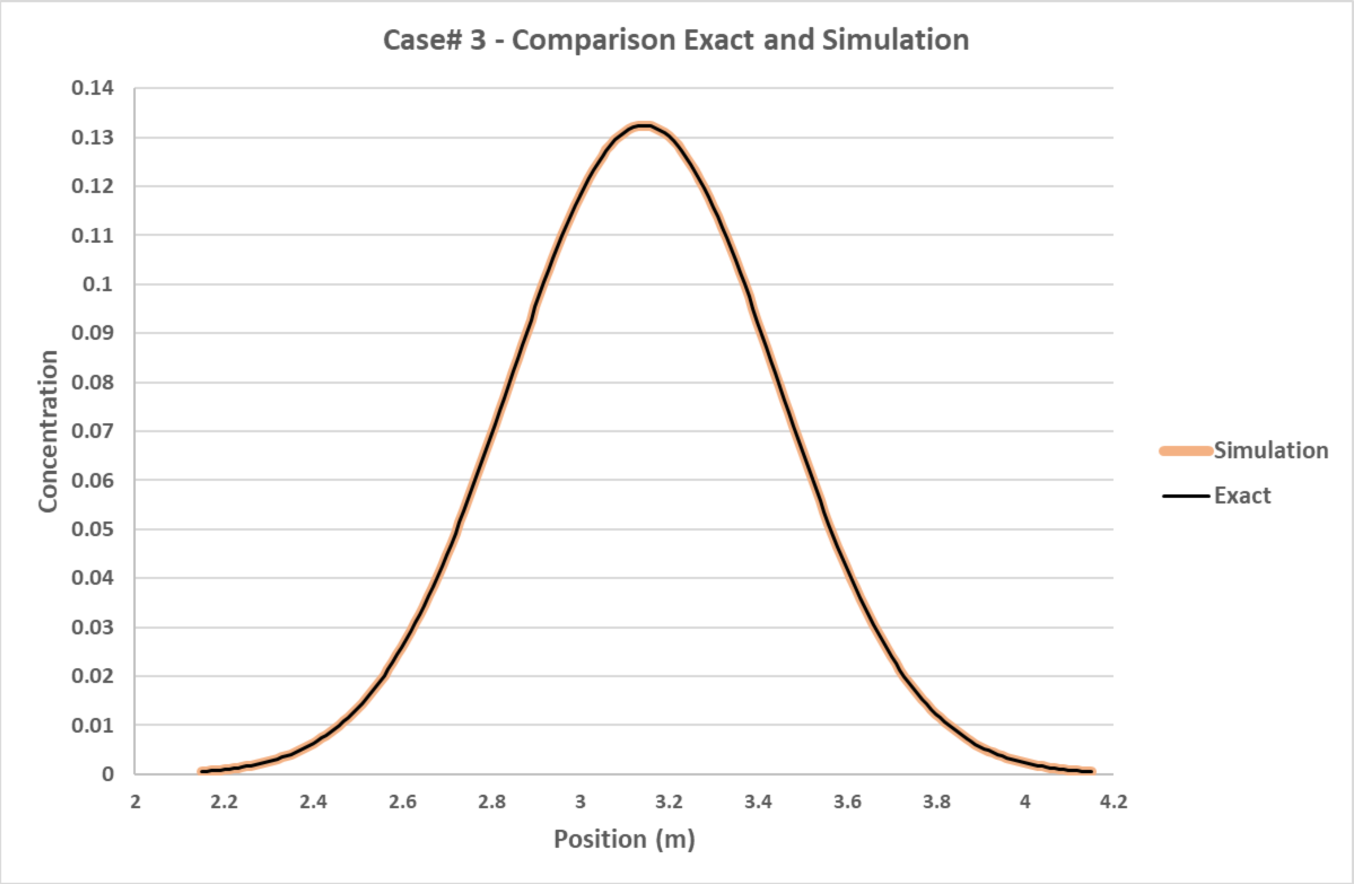
Diffusion coefficient according to Hart et al. (2016)

Follow tracer for 4 sec, until tracer center is at 3.15 m

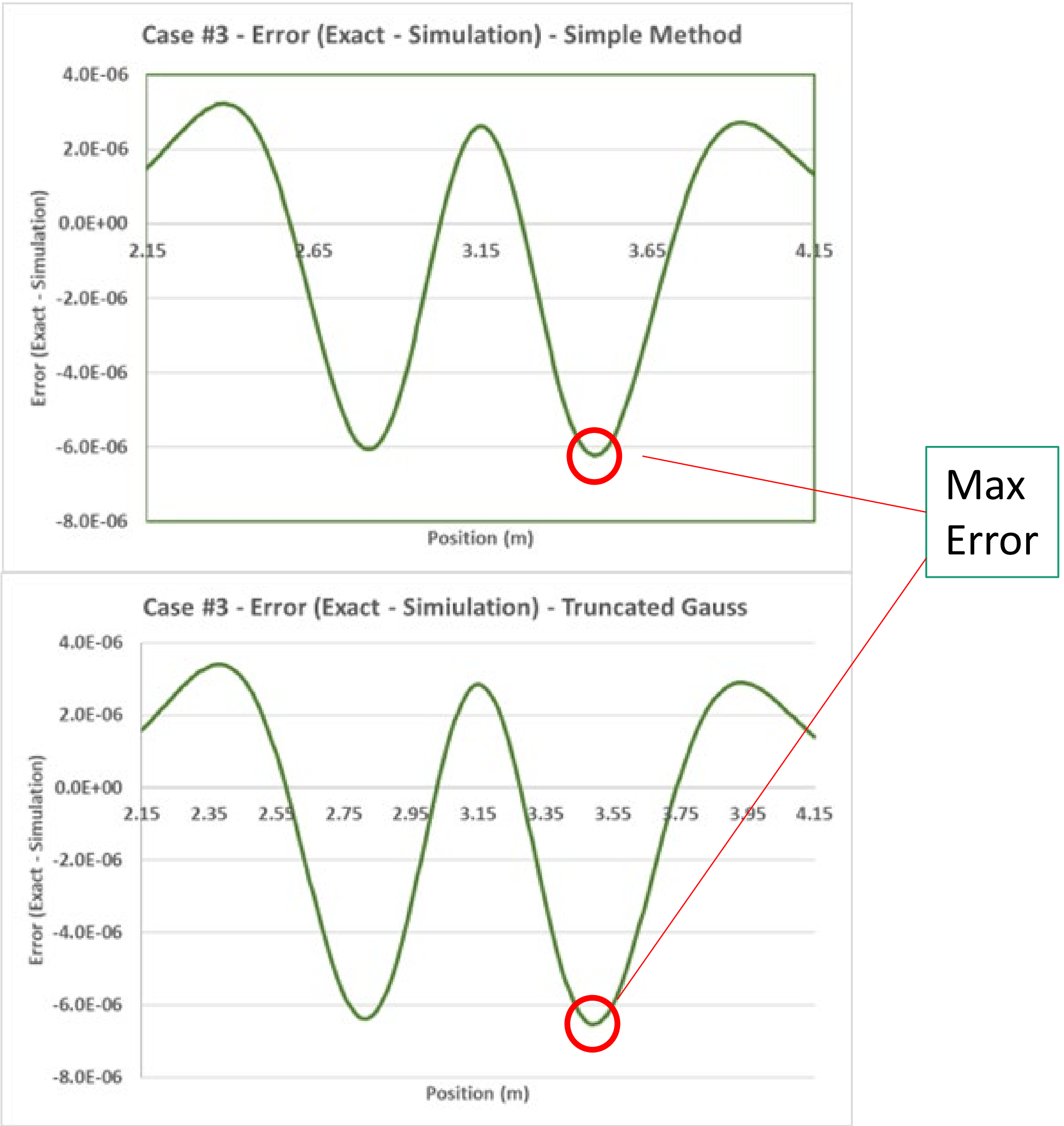
Seven different Cases

Case #	Δt	L	$U_s\ \Delta t/L$	TruncVal	NDmax
1	0.01 s	0.001 m	7.5	0.999	50
2	0.01 s	0.001 m	7.5	0.995	43
3	0.01 s	0.001 m	7.5	0.99	40
4	0.01 s	0.001 m	7.5	0.985	39/38
5	0.01 s	0.0005 m	15	0.99	81/80
6	0.005 s	0.0005 m	7.5	0.99	57
7	0.05 s	0.005 m	7.5	0.99	18

Compare numerical method to exact values



Subtract Simulation from Exact



"Standard" numerical solution for Advection-Diffusion Equation, 2 cases

- FTCS, Explicit **F**orward difference estimate for the **T**ime derivate (FT), and **C**entral difference approximation for the **S**pace derivatives (CS).

$$C_{new}(i) = \Delta t\ [D\ (C(i+1) - 2C(i) + C(i-1))/L^2 - U_s\ (C(i+1) - C(i-1))/(2\Delta t)]$$

- Stability criterion:

$$2\frac{D\Delta t}{L^2} \leq 1$$

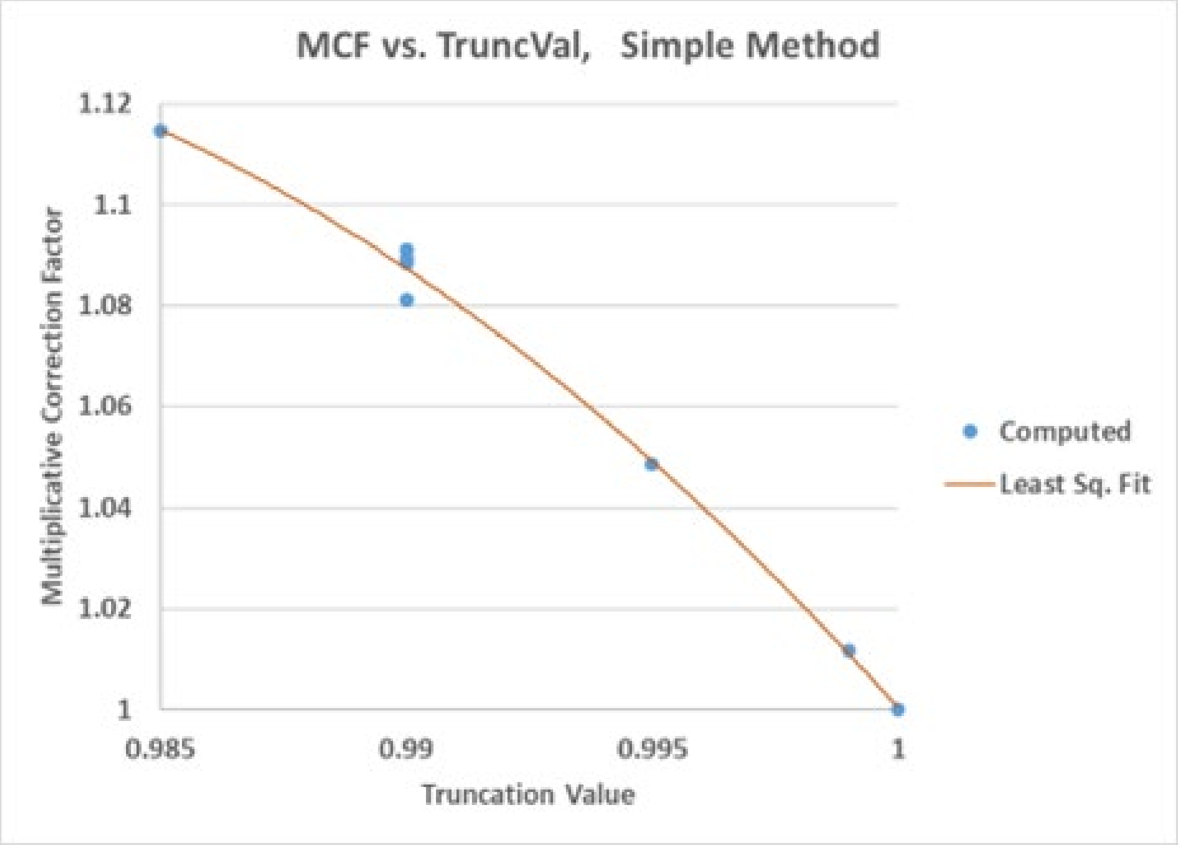
FTCS Case #	Δt	L
1	0.001 s	0.005 m
2	0.00004 s	0.001 m

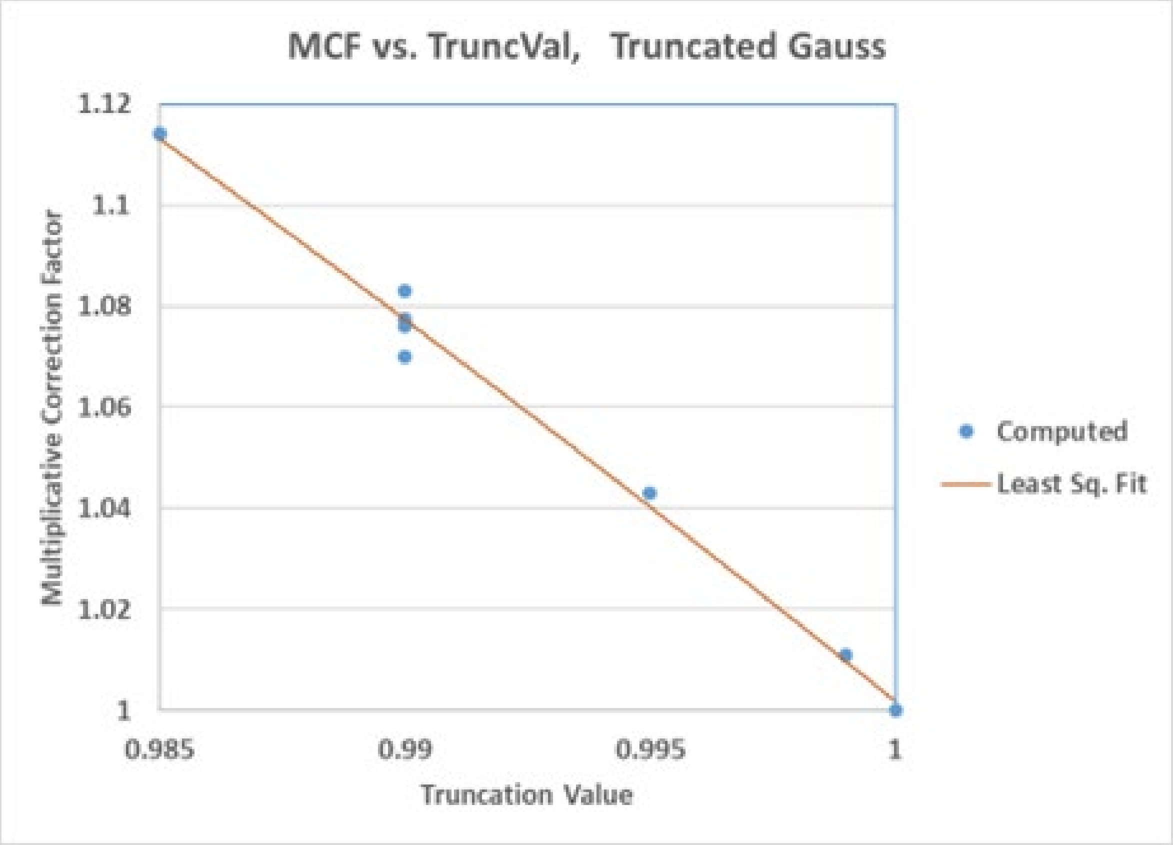
Results

Case	Trunc Val	Error SM	Error TG	Δt	L
#1	0.999	2.68×10^{-6}	2.23×10^{-6}	0.01 s	0.001 m
#2	0.995	4.8×10^{-6}	4.7×10^{-6}	0.01 s	0.001 m
#3	0.99	6.1×10^{-6}	6.4×10^{-6}	0.01 s	0.001 m
#4	0.985	47.0×10^{-6}	43.4×10^{-6}	0.01 s	0.001 m
#5	0.99	5.8×10^{-6}	6.4×10^{-6}	0.01 s	0.0005 m
#6	0.99	4.6×10^{-6}	3.6×10^{-6}	0.005 s	0.0005 m
#7	0.99	25.6×10^{-6}	26.6×10^{-6}	0.05 s	0.005 m
FTCS #1		1700.0×10^{-6}		0.001 s	0.005 m
FTCS #2		49.0×10^{-6}		0.00004 s	0.001 m

- Using Diffusion Operators produce lower or equivalent errors to the FTCS method
- Diffusion Operator method should use Truncation Values greater than 0.985
- Comparing Case #7 and FTCS #2:
  - Case #7 has roughly half the error of FTCS #2
  - Case #7 has a “box length” which is 5 times larger than that of FTCS #2
  - Case #7 has a time-step which is 1250 times larger than that of FTCS #2.

What about the Multiplicative Correction Factors?

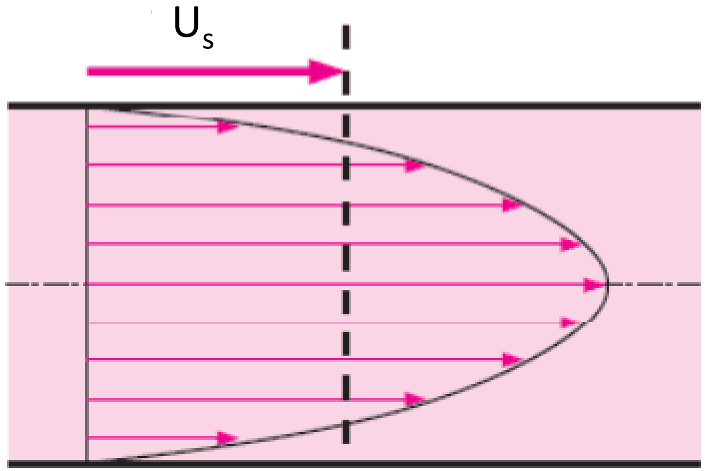




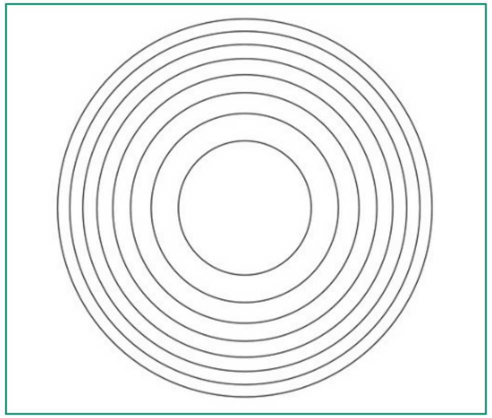
MCF is nicely dependent on Truncation Value. (2^{nd} order polynomial Least Sq. Fit)
A truncation of 1% reduces the Diffusion Constant with roughly 9%.

Dispersion in Laminar Flow

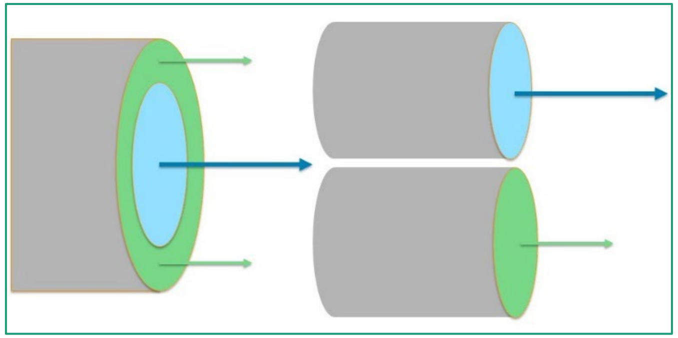
- Laminar flow: Fluid velocity vector “only” along the pipe (no radial component)
- Velocity profile: $U(r) = 2\,U_s\,(1 - r^2/R^2)$
- A small fluid “package” situated at r_0 will have a velocity $U(r_0)$ all along the pipe.



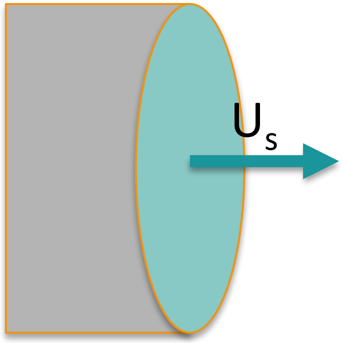
- Simulation strategy
  - Divide cross-sectional area into N equal sized sub-areas
  - Relative flux through each area: $RelFlux(i) = (2(N-i)+1)/N^2$
  - i = 1 is the innermost area

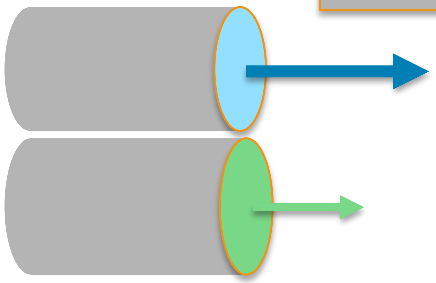


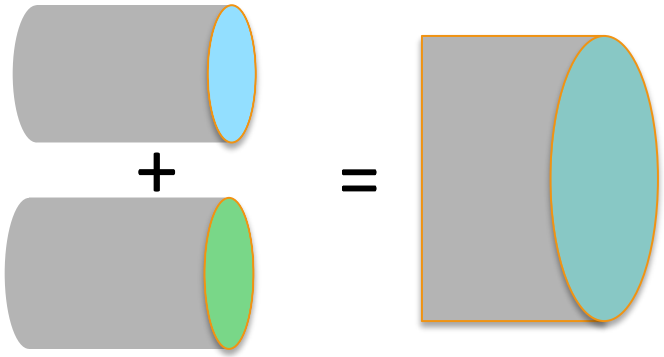
Dispersion in Laminar Flow   1-d ++ model



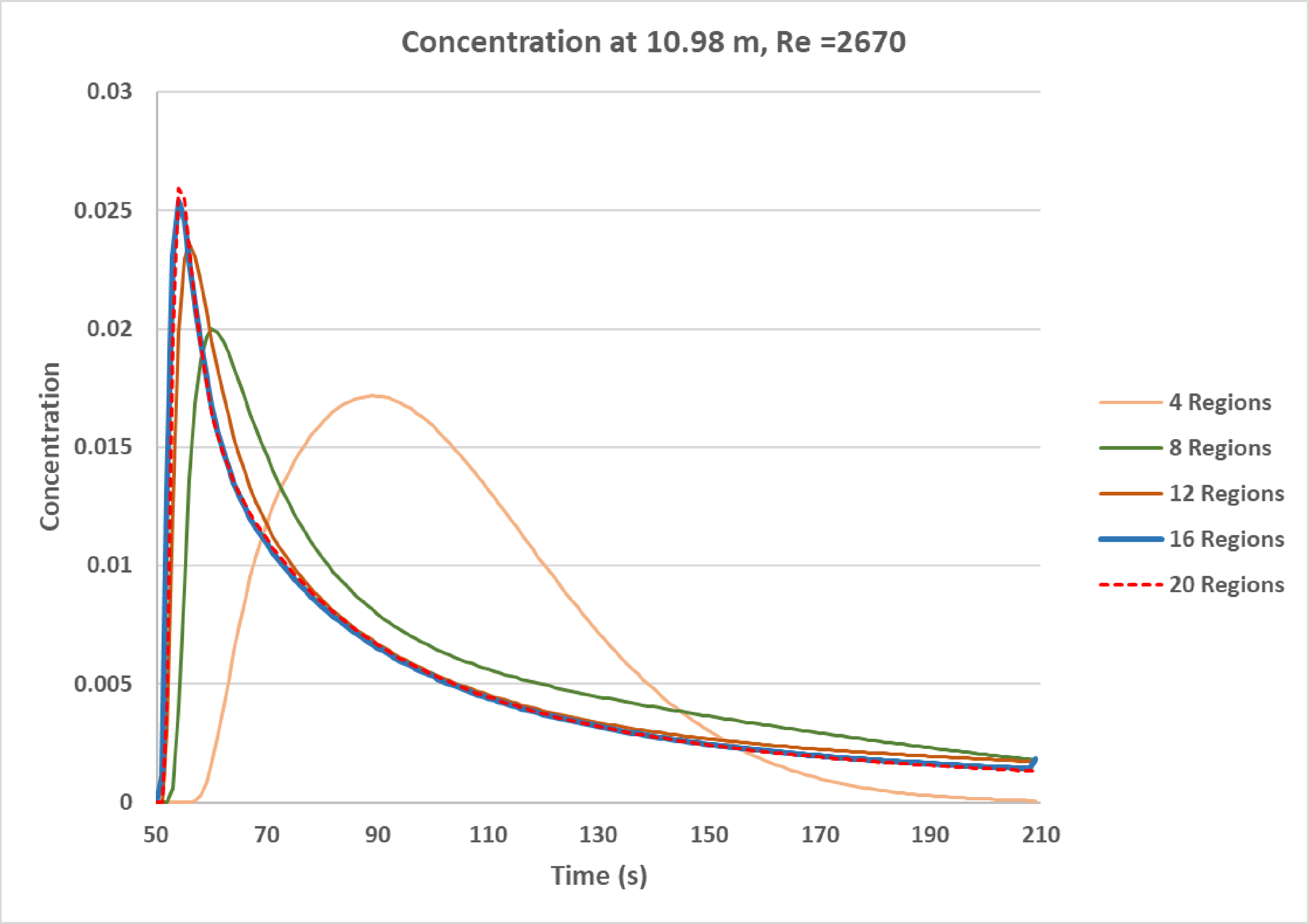
- N pipes of equal volumes
- Each 1/N volume of the "main" pipe
- Solve one time-step Convection of main pipe
  - Pressure computed in every numerical box
- Do displacement for each sub-pipe
  - $\mathrm{RelFlux}(i) = (2(N-i)+1)/N^2$
  - Small radial dispersion between neighbor pipes
- New "main" pipe = sum of sub-pipes
- Solve on time-step Convection of updated main pipe
- Etc.



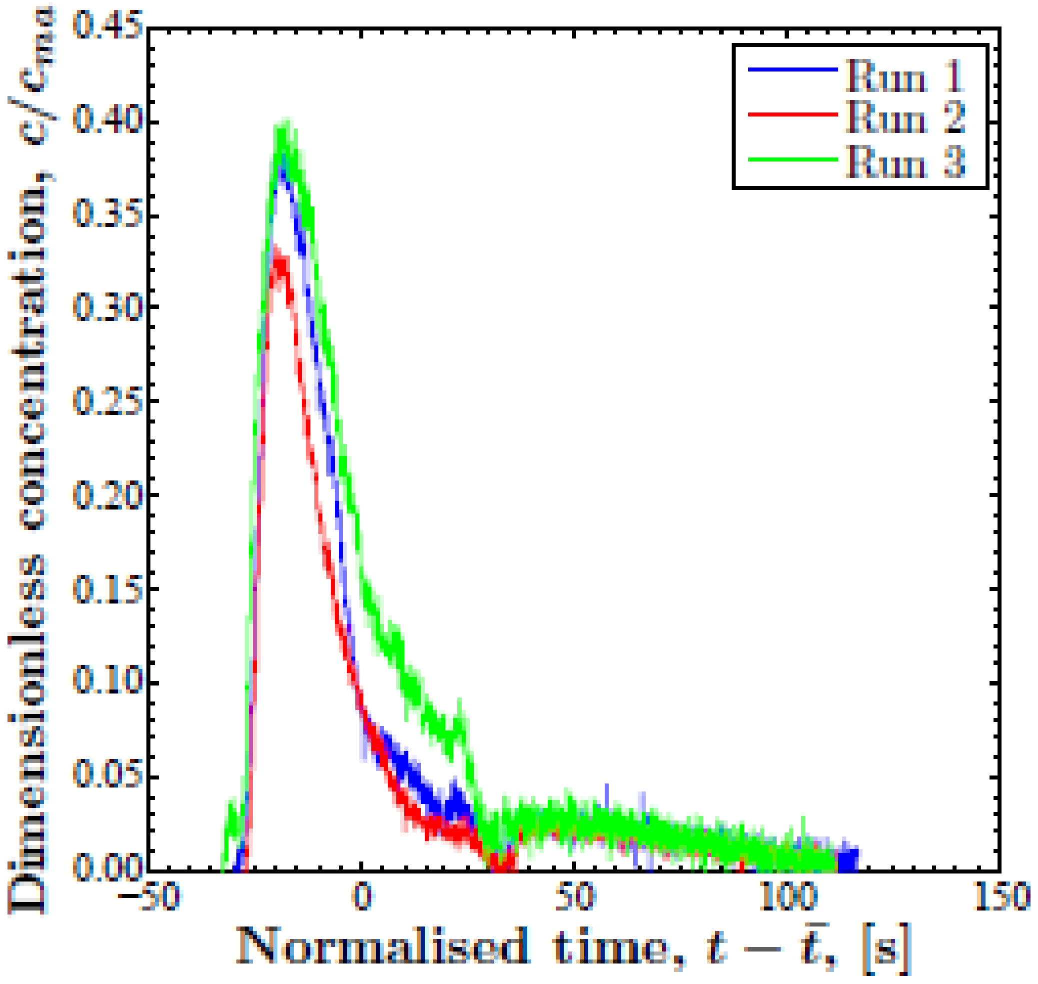




Dispersion with Laminar Flow

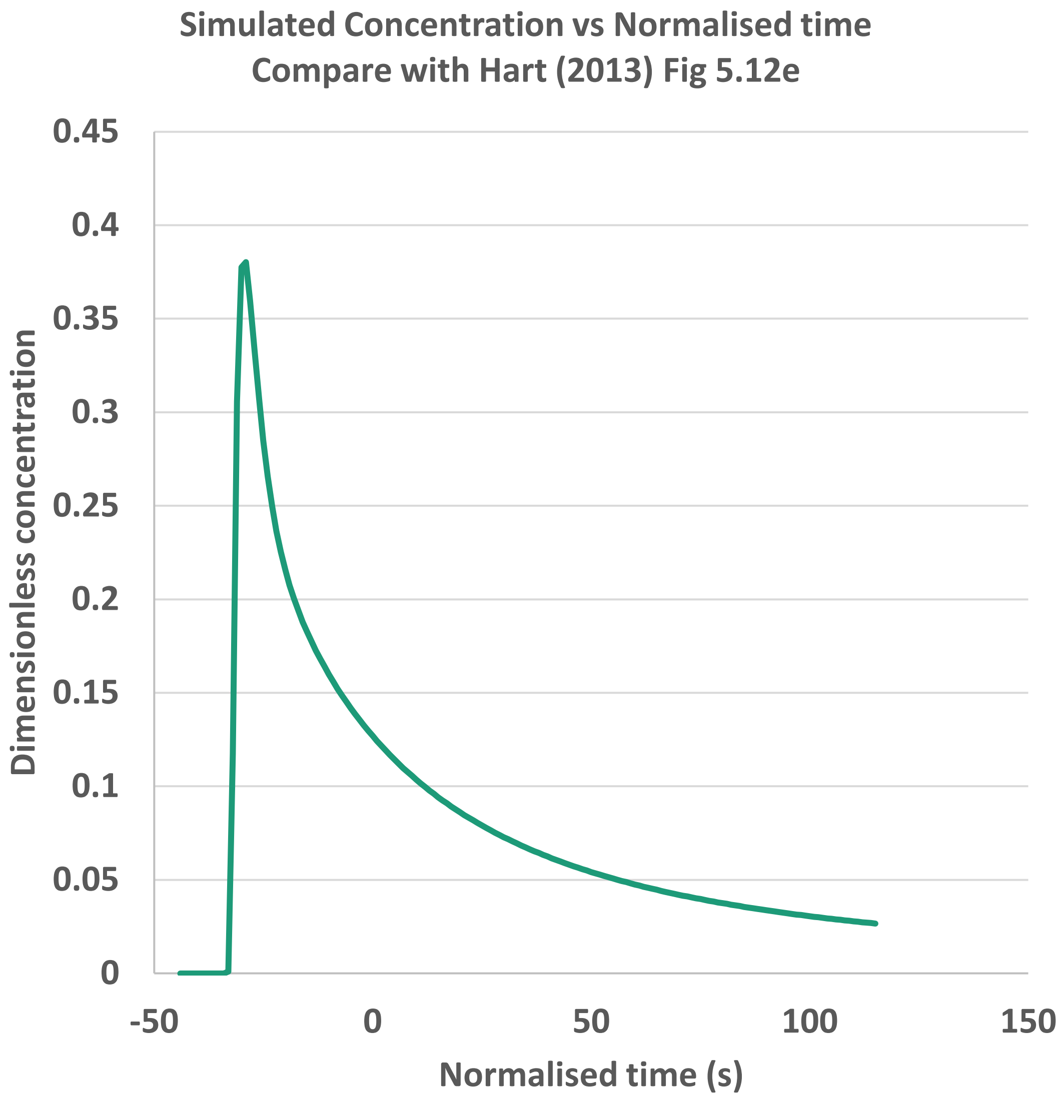


Comparison with James Hart’s experiment



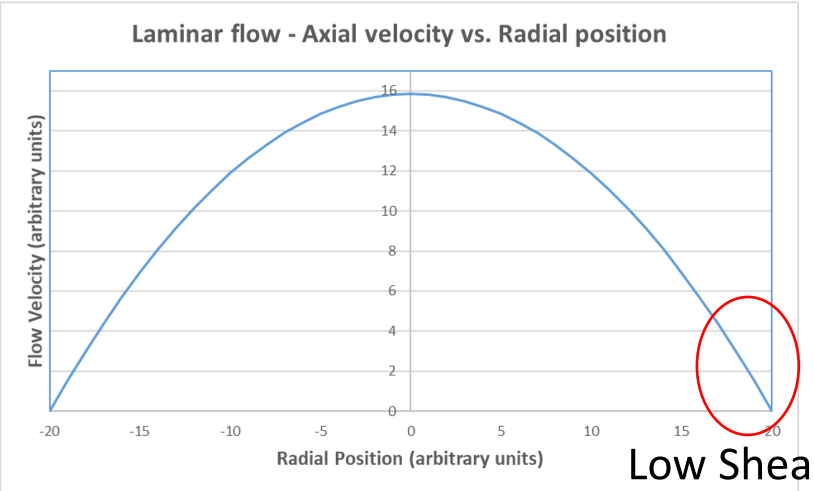
(e) Re = 2670.

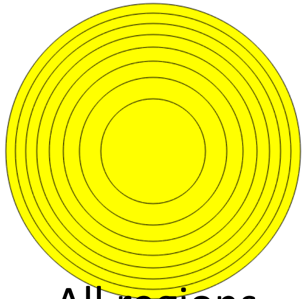
Figure courtesy James Hart



Dispersion in Laminar-Turbulent Transitional Flow

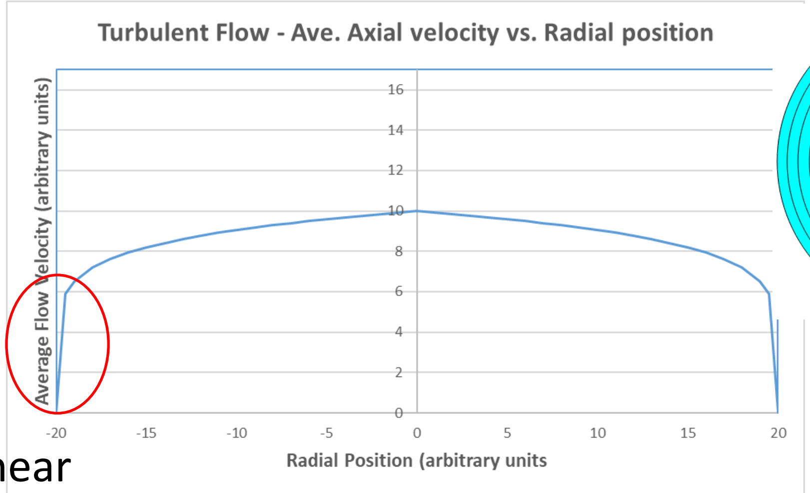
Very simplified approach: Gradual "flattening" at center of average velocity profile.

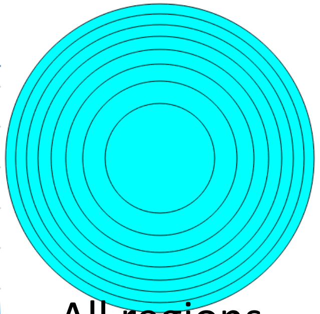




All regions
Laminar

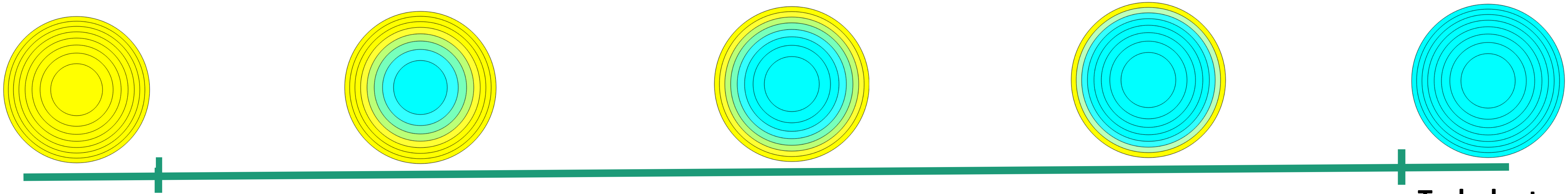
Low Shear
Low Friction Factor





All regions
Turbulent

High Shear
High Friction Factor



Laminar

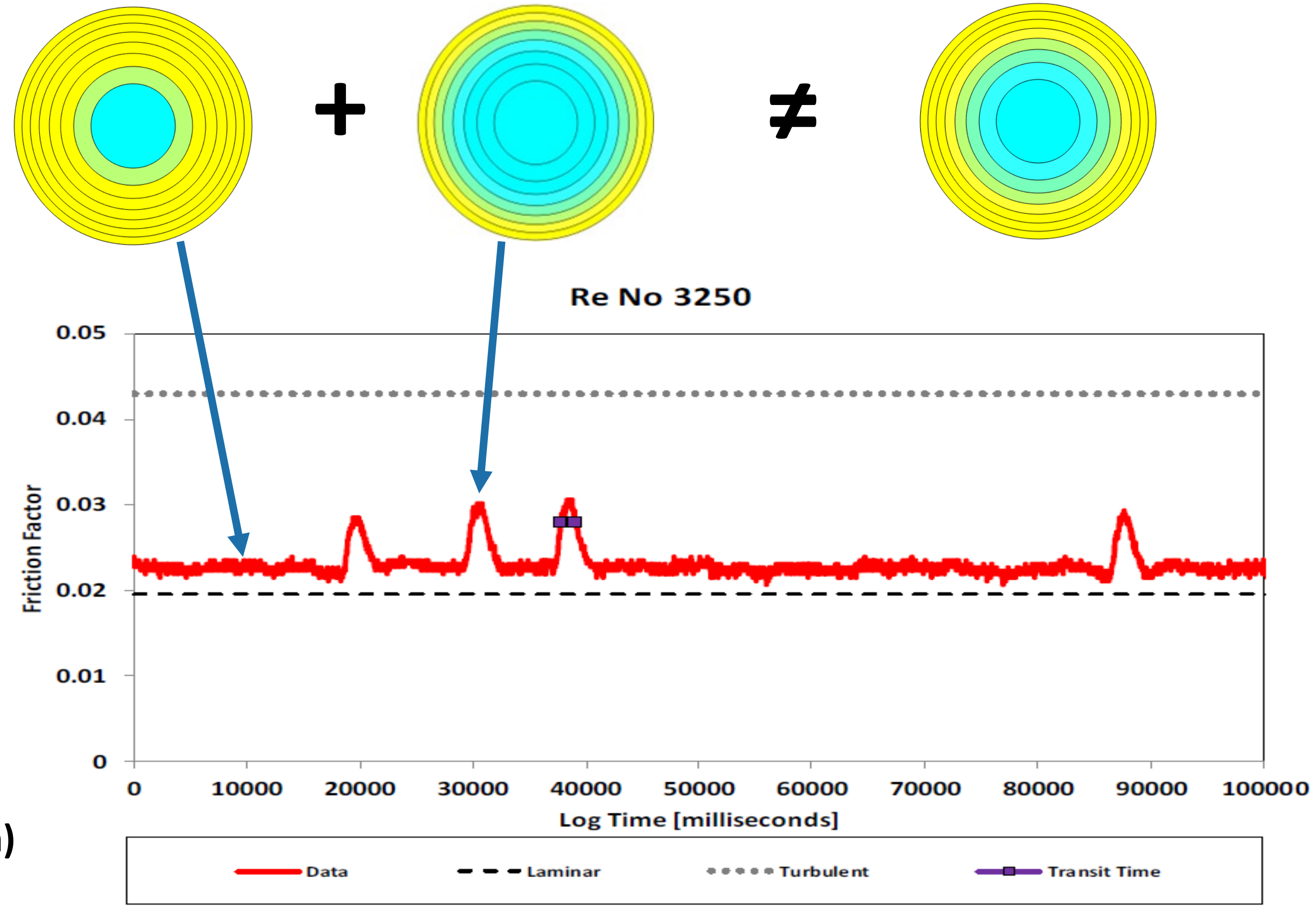
Increasing Re

Turbulent

Dynamically, Dr. C.Mills* work doesn't support this:

Near laminar flow interrupted by turbulent bursts.

Dynamic Simulation must include "random" bursts (in time).



Used by permission from Dr. Christopher Mills.

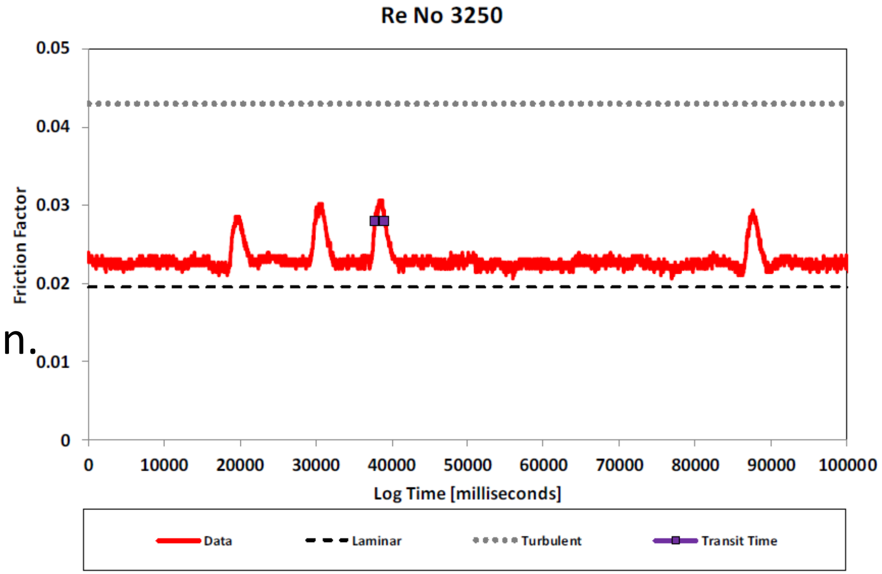
Laminar Dispersion Only

Turbulence (Axial & Radial Dispersion)

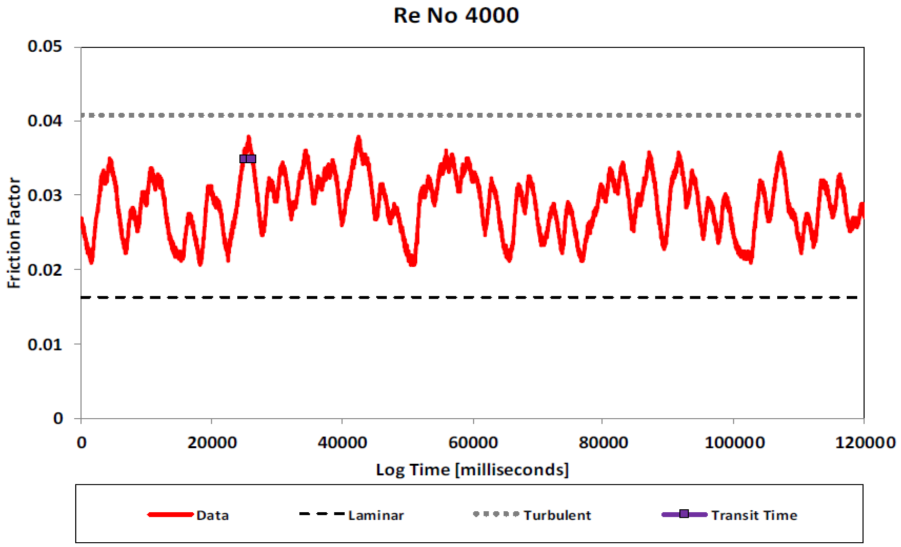
*Mills, Christopher, 2020, "Identifying the transition between laminar and turbulent flow using high-frequency pressure loss measurements", Coventry University Engineering Doctorate Thesis, May 2020, https://pureportal.coventry.ac.uk/en/studentTheses/identifying-the-transition-between-laminar-and-turbulent-flow-usi

Dispersion in Transitional Flow

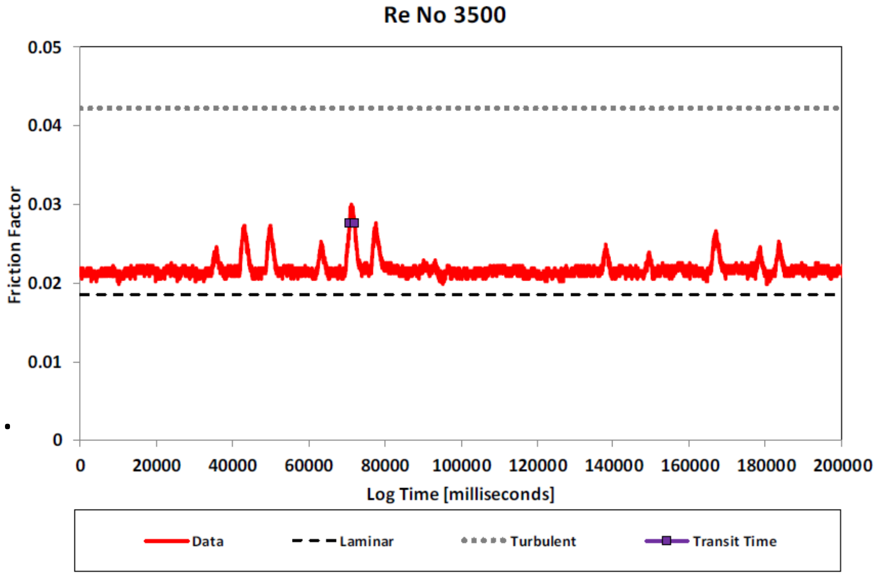
Turbulence mostly at center. Bursts widen the radial dispersion.



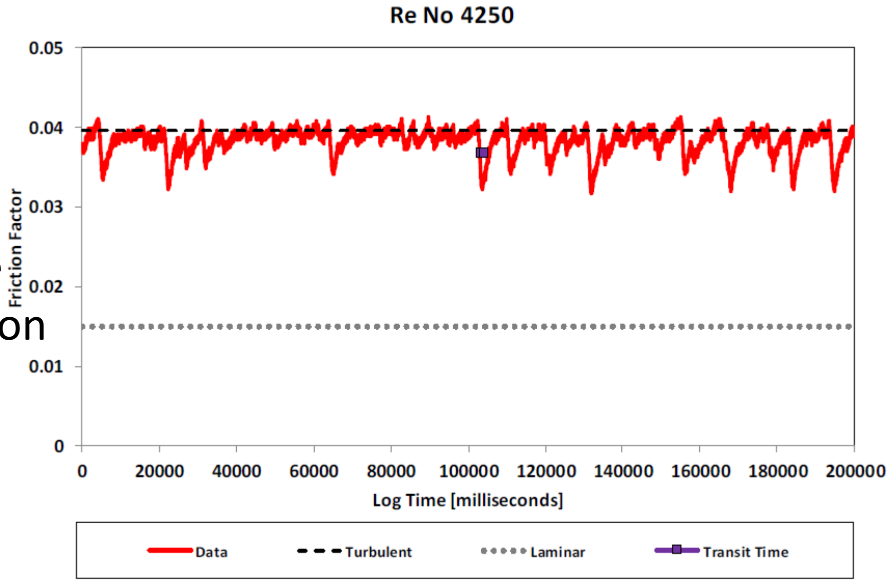
Turbulence width random.



Turbulence mostly at center. Bursts widen the radial dispersion.



Turbulence in the whole cross-section most of the time.



Conclusions

- **Advection-Diffusion Dynamic Simulator (Dispersion with Turbulent Flow)**
  - **Fast**
  - **Accurate**
  - **Flexible**
- **Laminar Flow**
  - **Promising results**
  - **Fits in same numerical frame-work as Dispersion with Turbulent Flow**
- **Transitional Flow**
  - **Work needs to be done**
  - **Fits in same numerical frame-work as Laminar & Turbulent Flow**

When Transitional Flow simulator has been "verified", this can become: Dynamical Simulator that will in run-time pass between Laminar, Transitional and Turbulent Flow as flow rates change and valves open and shut.

Questions/Comments?