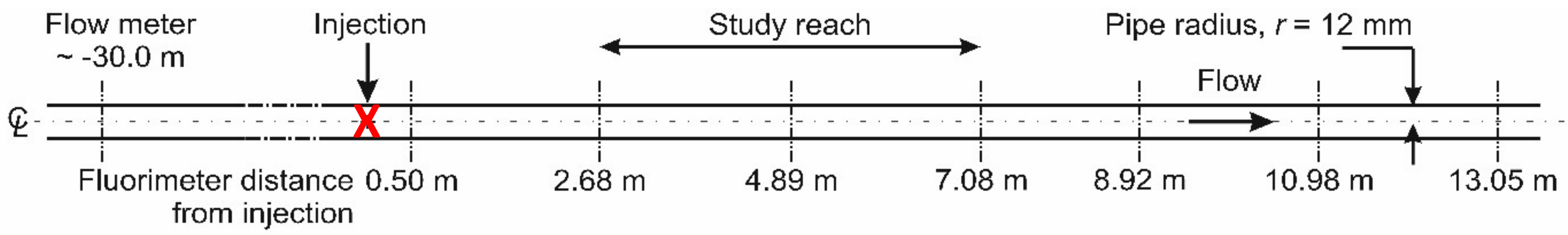
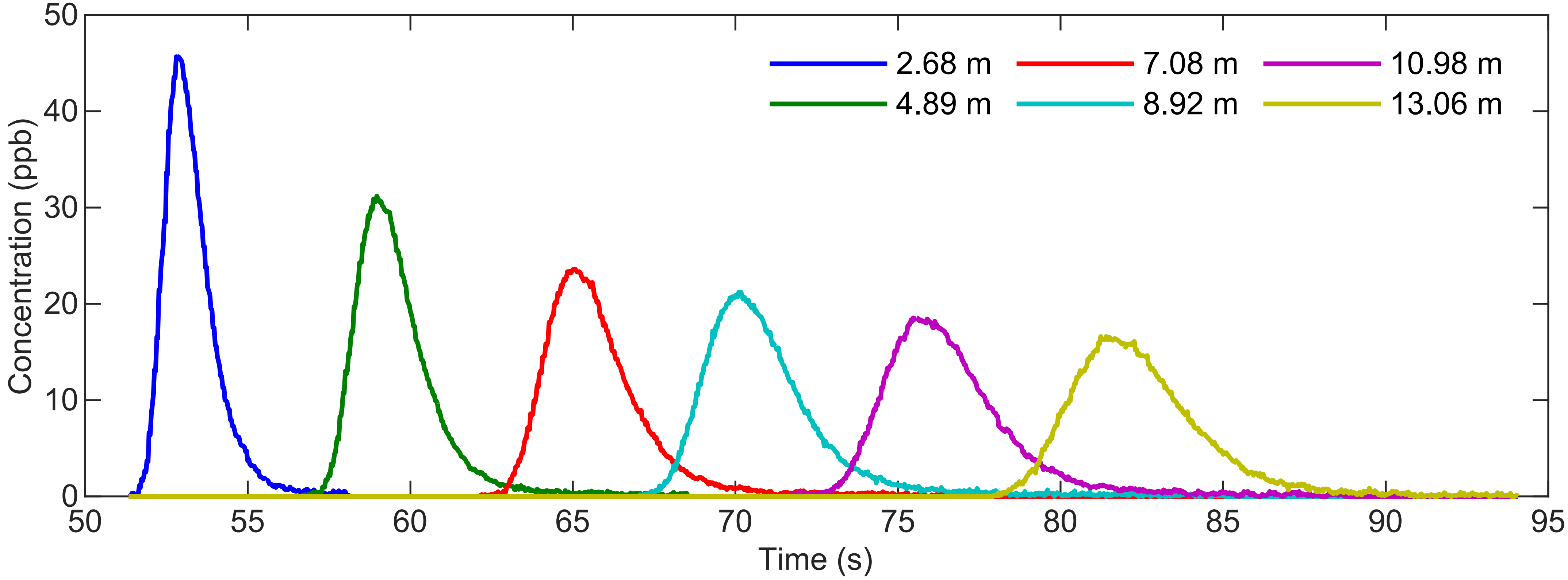
Longitudinal Dispersion in Pipes for Steady and Unsteady Flows

James Hart

Coventry University

Longitudinal Dispersion in Pipes

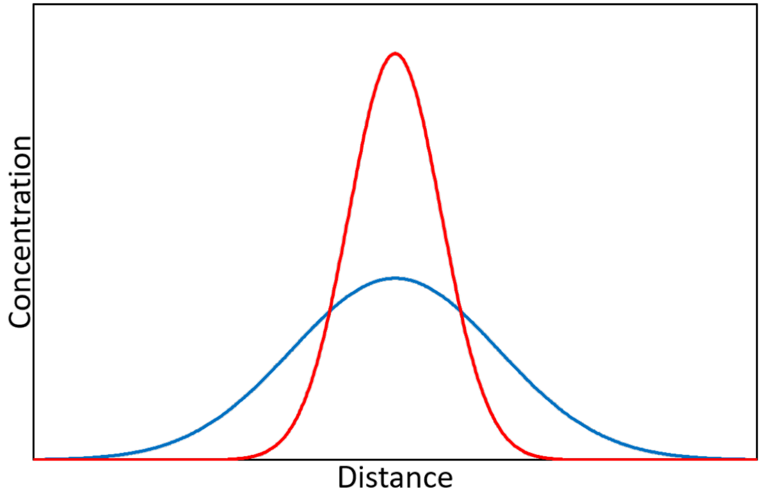




G. I. Taylor (1953, 1954)

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law:

$$\boxed{c(x,\bar{t}) = \frac{1}{\sqrt{4\pi D_{xx}\bar{t}}}\,exp\left(-\frac{(x-u\bar{t})^2}{4D_{xx}\bar{t}}\right)}$$



2. Taylor derived two equations for D_{xx}:

Laminar Flow (Re < 2000):

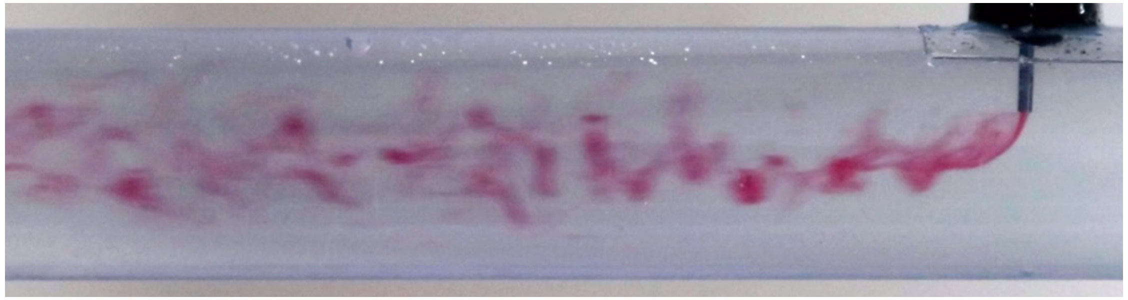
$$D_{xx} = r^2u^2/48D_m$$

Turbulent Flow (Re > 4000):

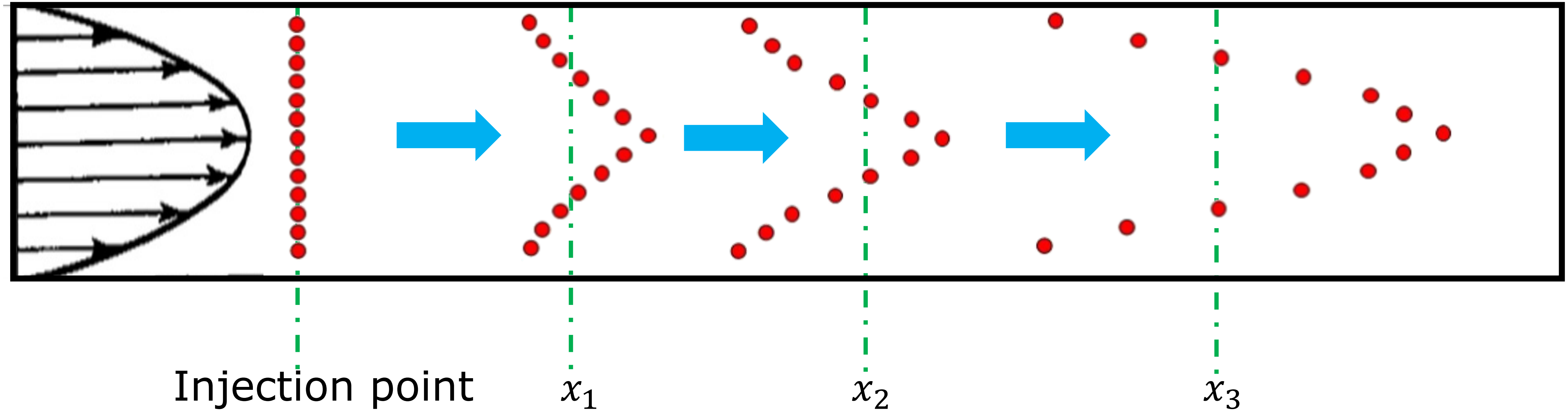
$$D_{xx} = 10au_*$$

Mixing Mechanisms

Mixing mechanisms:



- Molecular diffusion (10^{-10} to 10^{-9} m^2/s)
- Turbulent diffusion (10^{-3} to $10^{-1} m^2/s$)
- Differential advection (1 to 10^3 m^2/s)

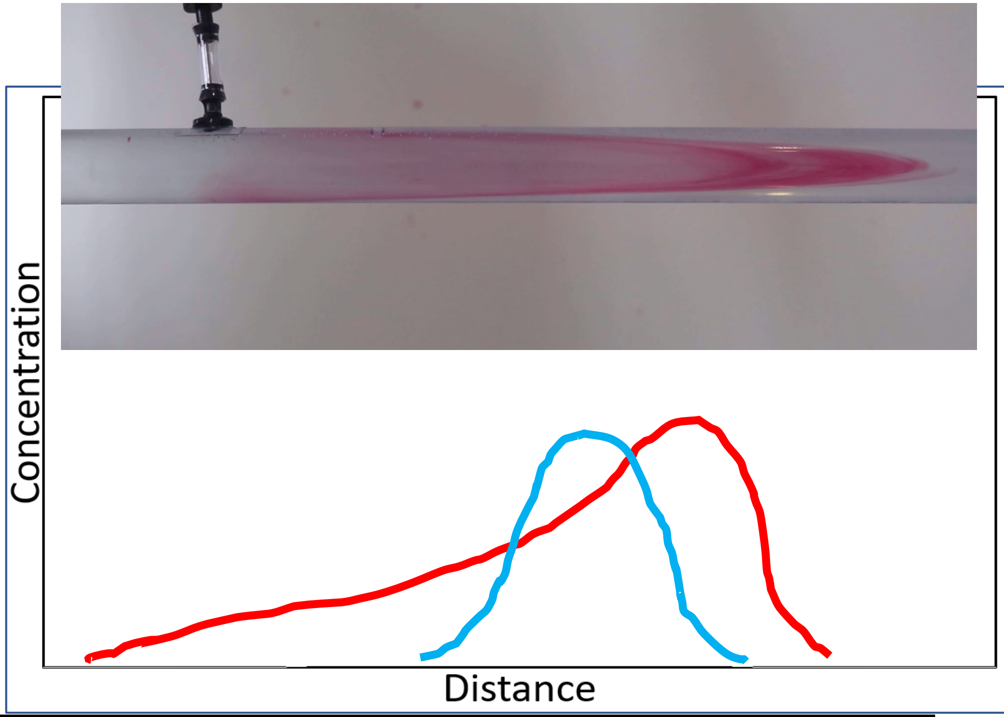


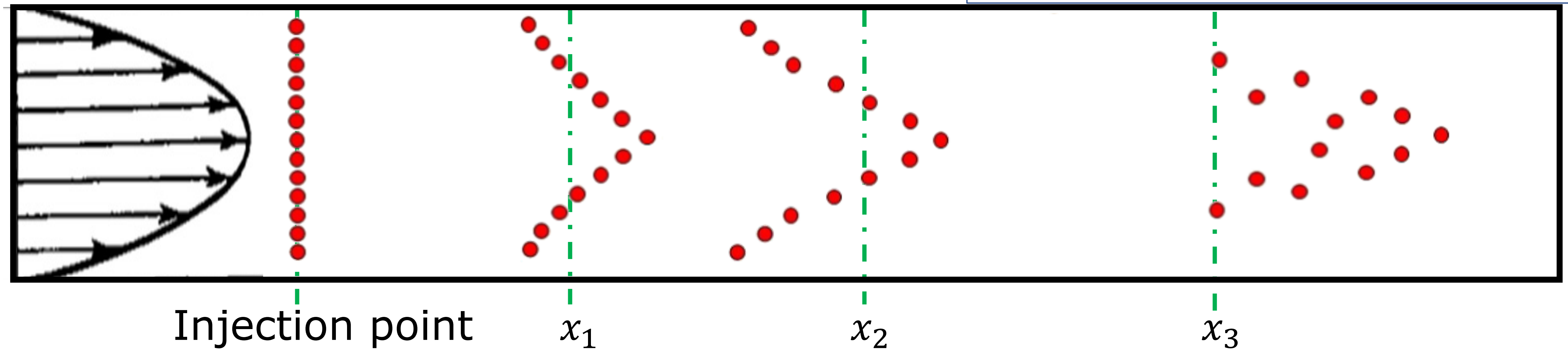
Mixing Mechanisms

Mixing Mechanisms

Mixing mechanisms:

- Molecular diffusion (10^{-10} to 10^{-9} m^2/s)
- Turbulent diffusion (10^{-3} to $10^{-1} m^2/s$)
- Differential advection (1 to 10^3 m^2/s)





G. I. Taylor (1953, 1954)

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law:

$$c(x,\bar{t}) = \frac{1}{\sqrt{4\pi D_{xx}\bar{t}}} exp\left(-\frac{(x-u\bar{t})^2}{4D_{xx}\bar{t}}\right)$$

Taylor derived two equations for D_{xx}:

Assumes flow is highly turbulent (Re > 20,000)!

All this is for steady flow!

Laminar Flow (Re < 2000):

$$D_{xx} = r^2u^2/48D_m$$

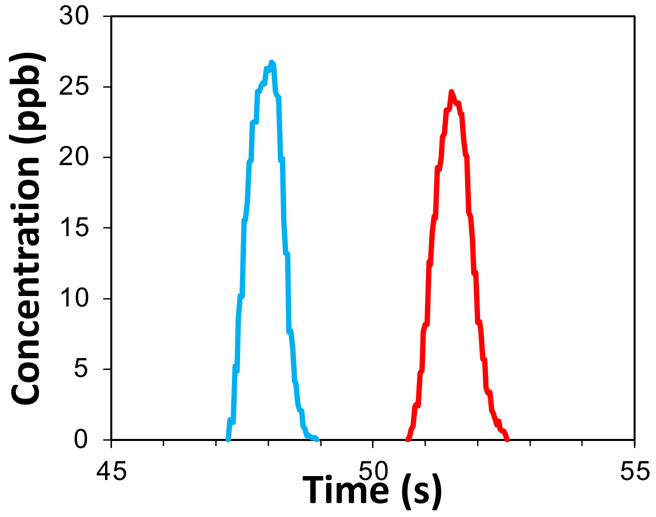
Turbulent Flow ~~(Re > 4000)~~:

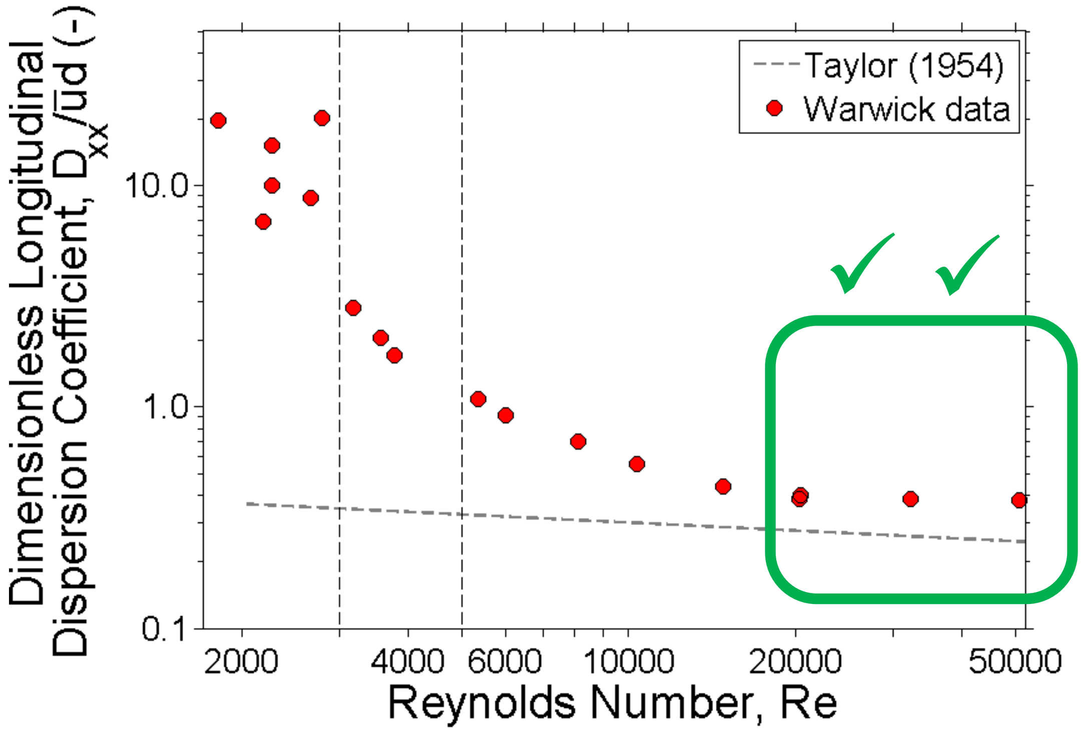
$$D_{xx} = 10au_*$$

Steady Tests: 2000 < Re < 20,000

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law

2. Taylor derived two equations for D_{xx}

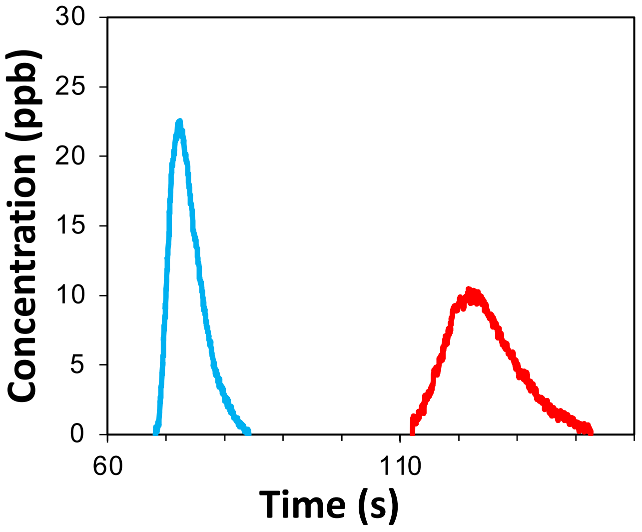


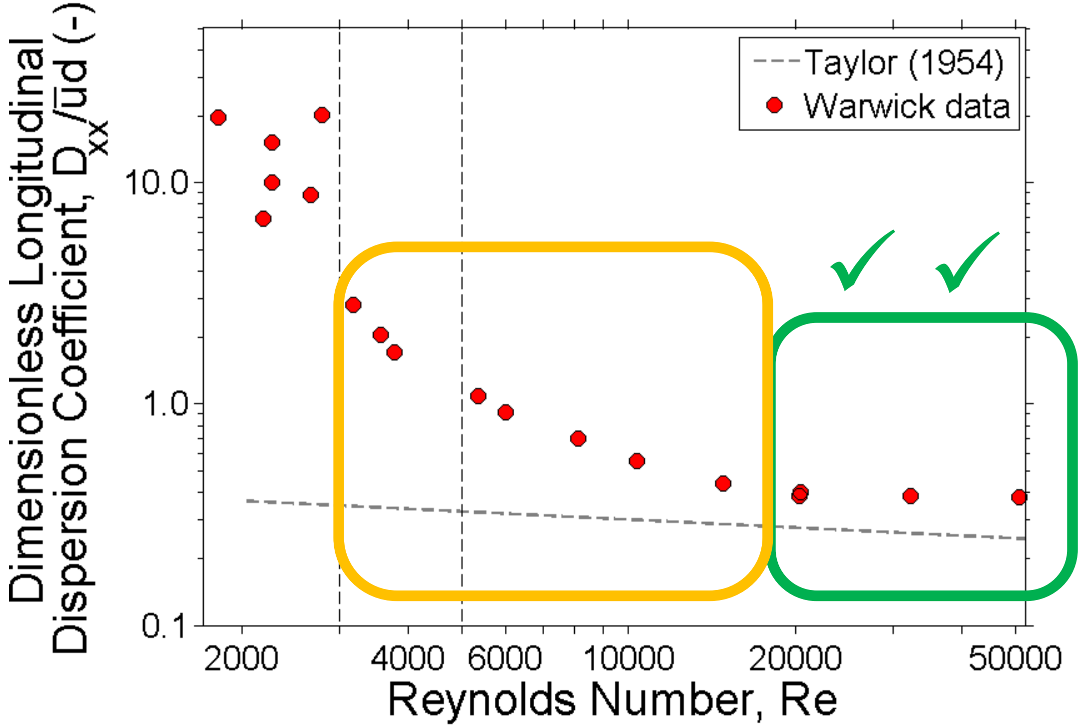


Steady Tests: 2000 < Re < 20,000

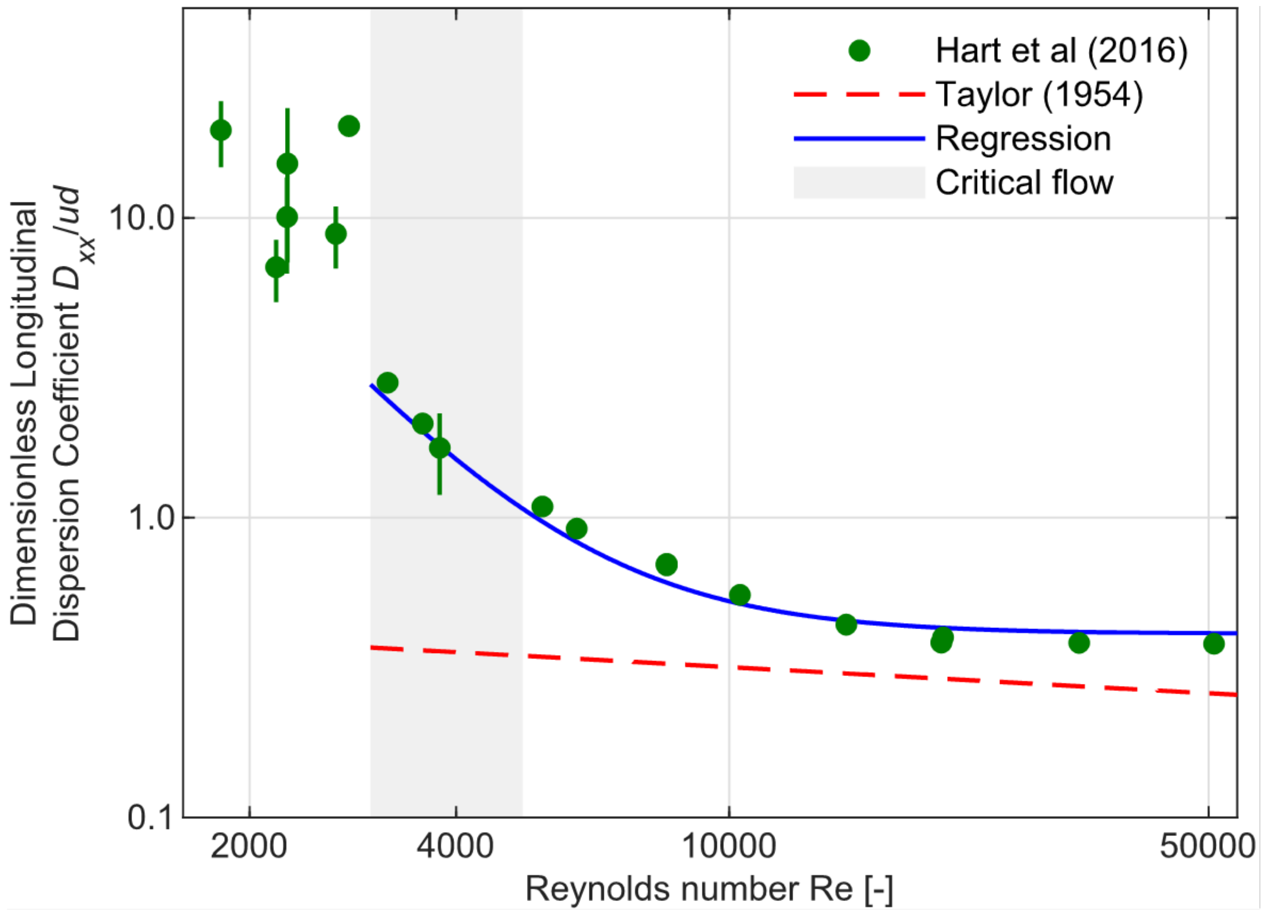
1. Concentration profiles are Gaussian (a certain time after injection) ✗ and described by Fick's second law

2. Taylor derived two equations for D_{xx} ✗





Steady Tests: 2000 < Re < 20,000



Hart et al. (2016):

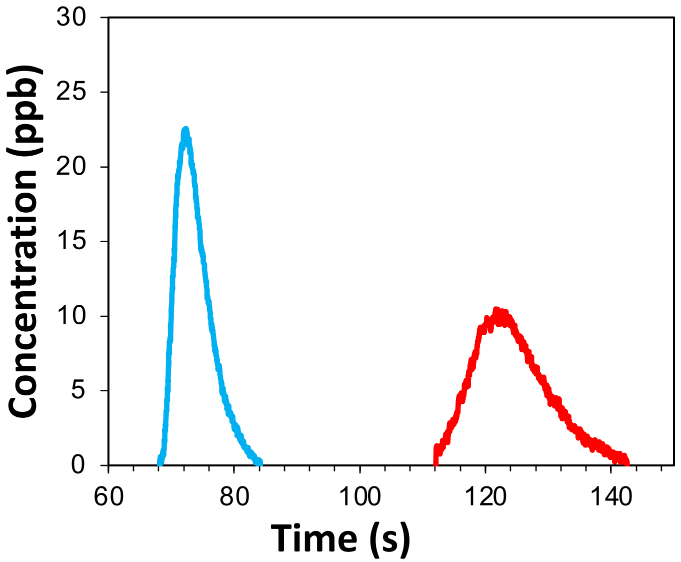
$$\frac{D_{xx}}{ud} = 1.17 \times 10^9 Re^{-2.5} + 0.41$$

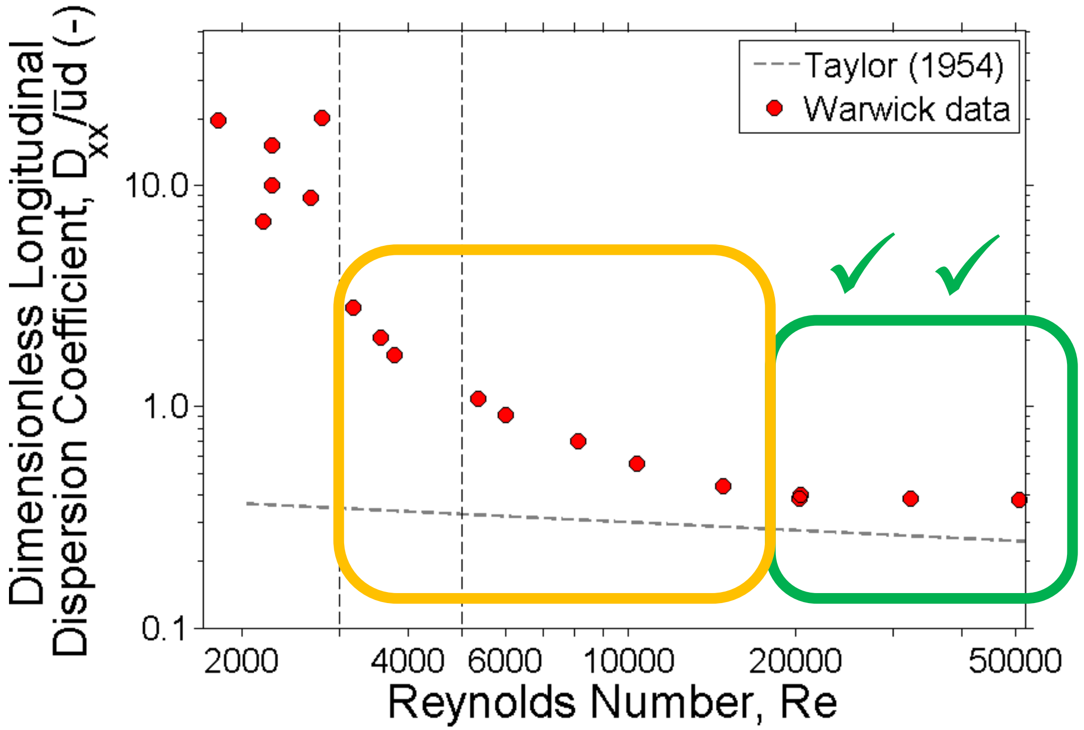
Steady Tests: 2000 < Re < 20,000

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law

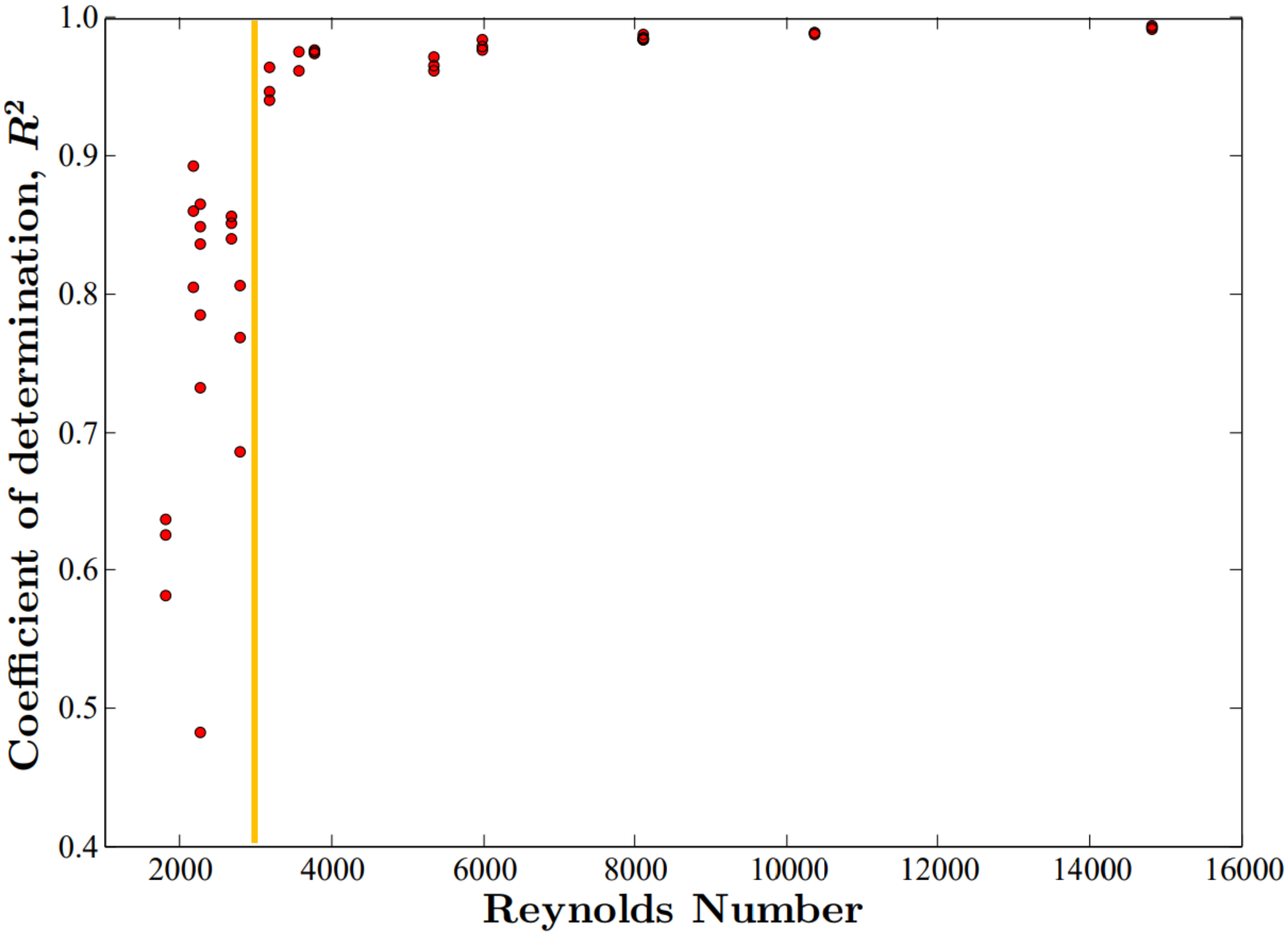
2. Taylor derived two equations for D_{xx}

- **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}





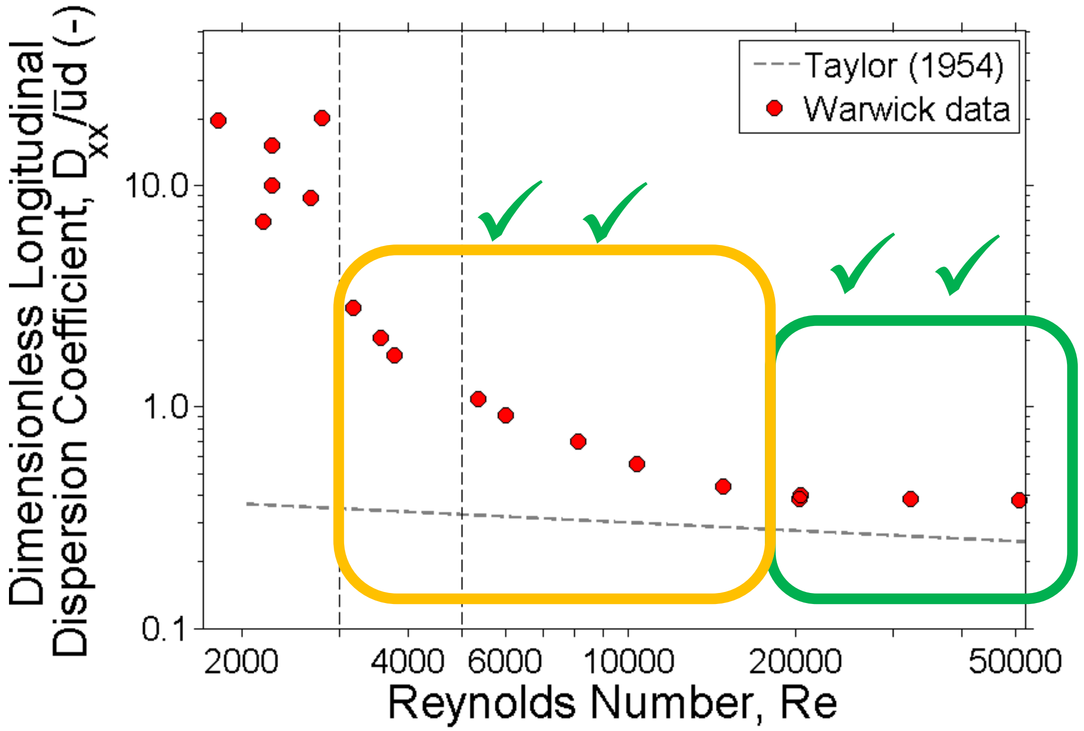
Steady Tests: 2000 < Re < 20,000



Steady Tests: 2000 < Re < 20,000

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law
   - **Turbulent flow:** Gaussian transfer function is reasonable

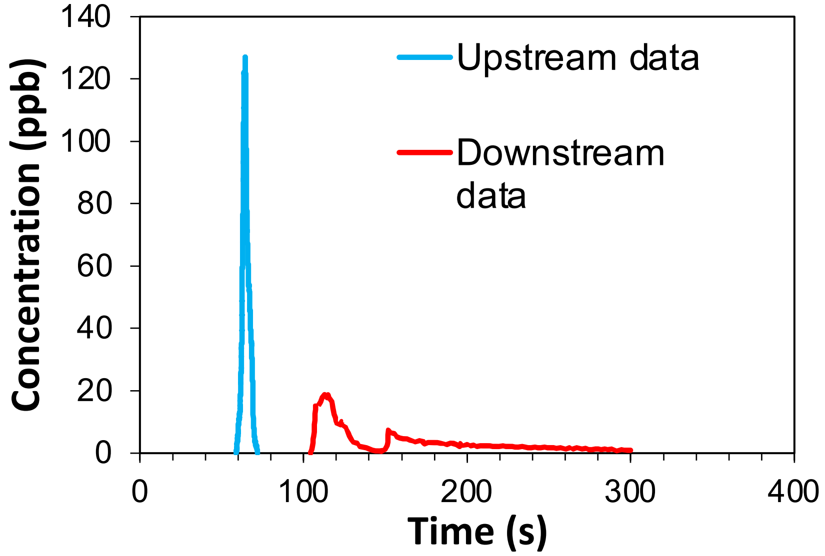
2. Taylor derived two equations for D_{xx}
   - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}

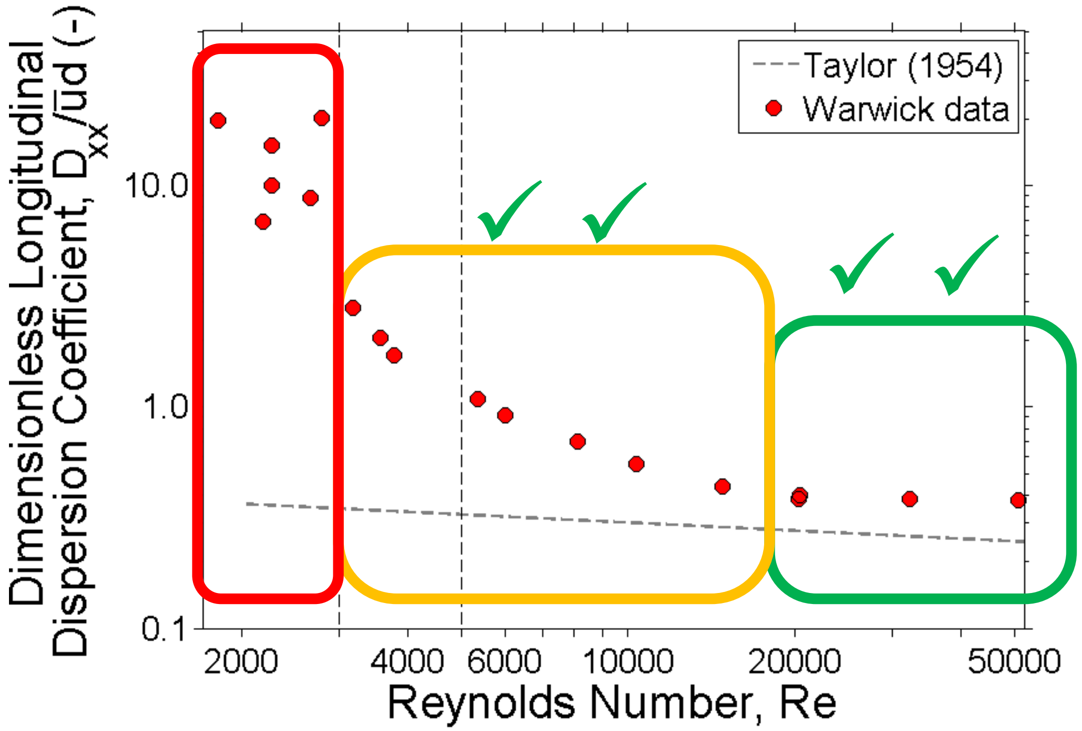


Steady Tests: 2000 < Re < 20,000

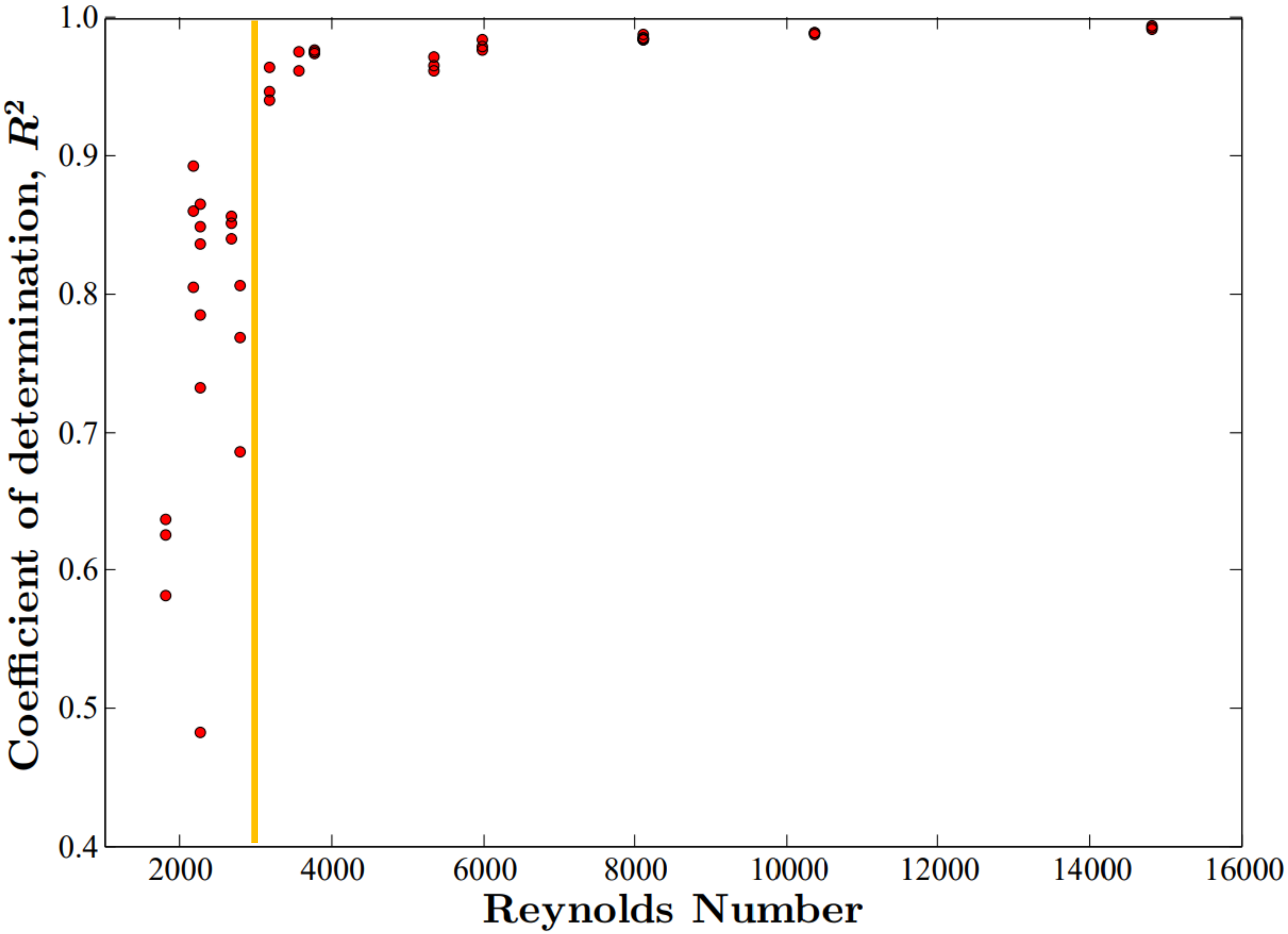
1. Concentration profiles are Gaussian <u>(a certain time after injection)</u> ✗ and described by Fick's second law
   - **Turbulent flow:** Gaussian transfer function is reasonable

2. Taylor derived two equations for D_{xx} ✓
   - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}





Steady Tests: 2000 < Re 20,000



Gaussian ‘a certain time after injection’

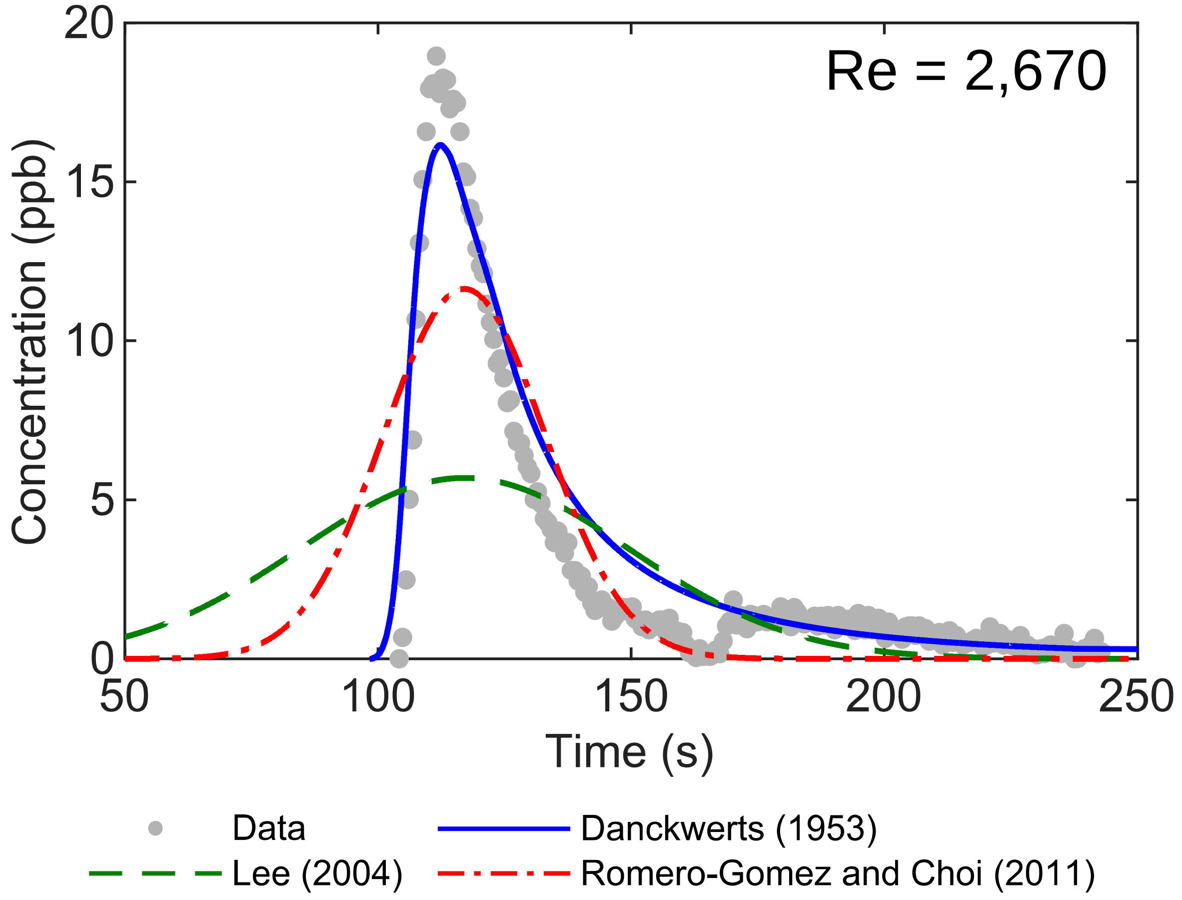
$$\tau = \frac{D_r t}{a^2} > 0.5$$

- Turbulent flow ≈ 2 seconds
- Laminar flow ≈ 2 weeks

CONTINUOUS FLOW SYSTEMS. DISTRIBUTION OF RESIDENCE TIMES

P. V. DANCKWERTS
Department of Chemical Engineering, Tennis Court Road, Cambridge, England, U.K.

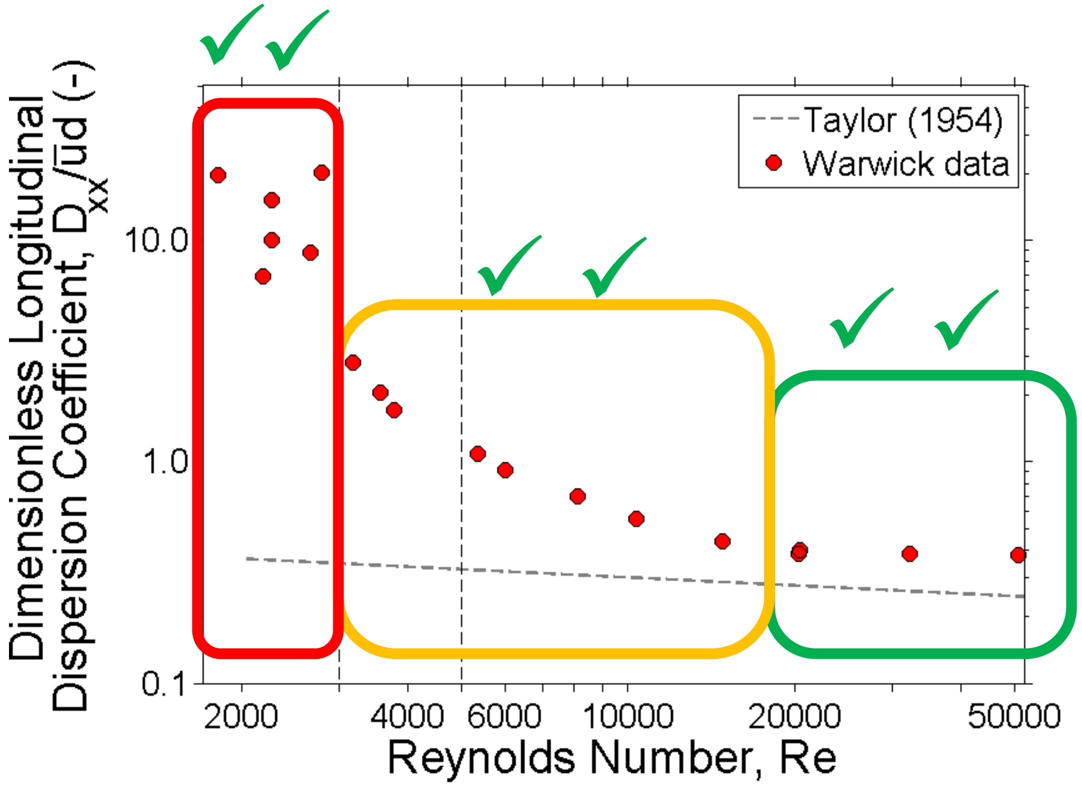
(Received 24 October 1952)



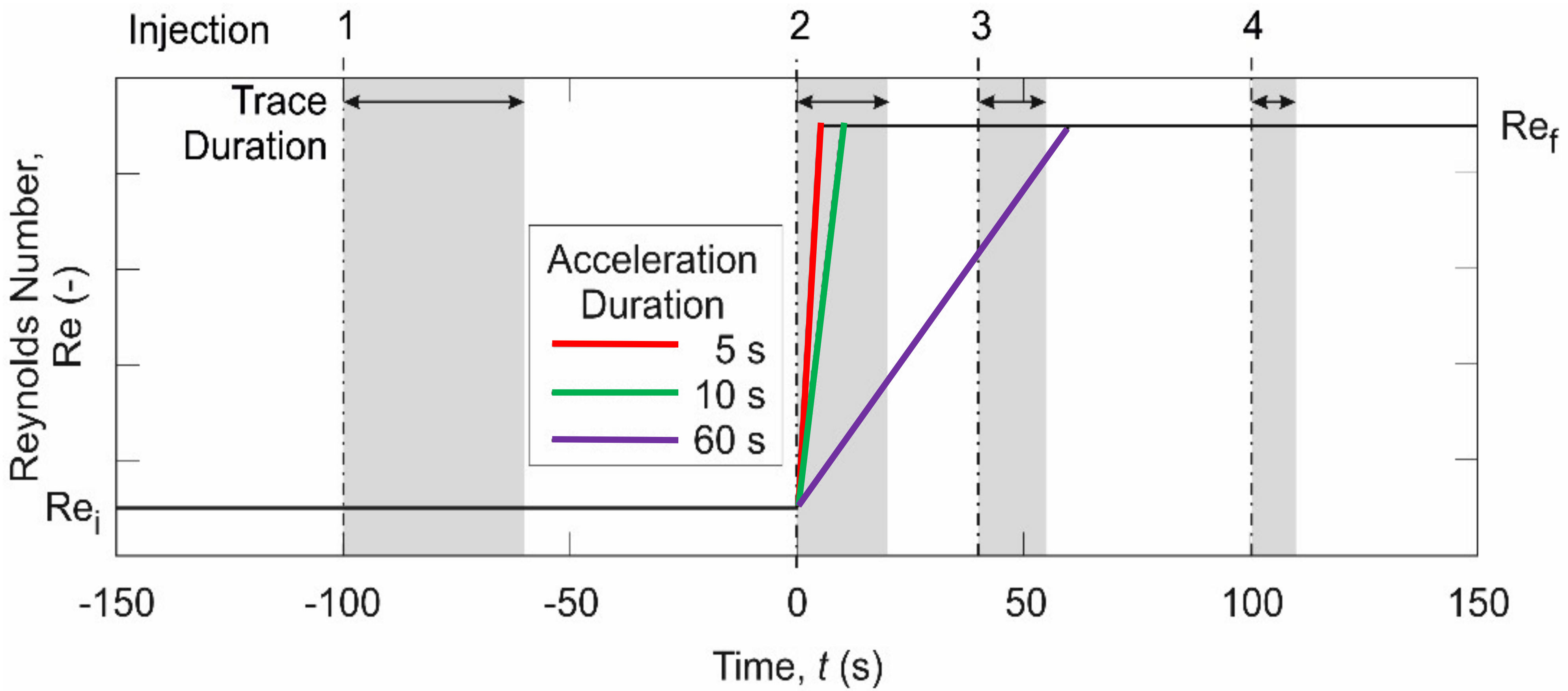
Steady Tests: 2000 < Re < 20,000

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law
    - **Turbulent flow:** Gaussian transfer function is always reasonable
    - **Laminar Flow:** At small times from injection, assume no radial mixing

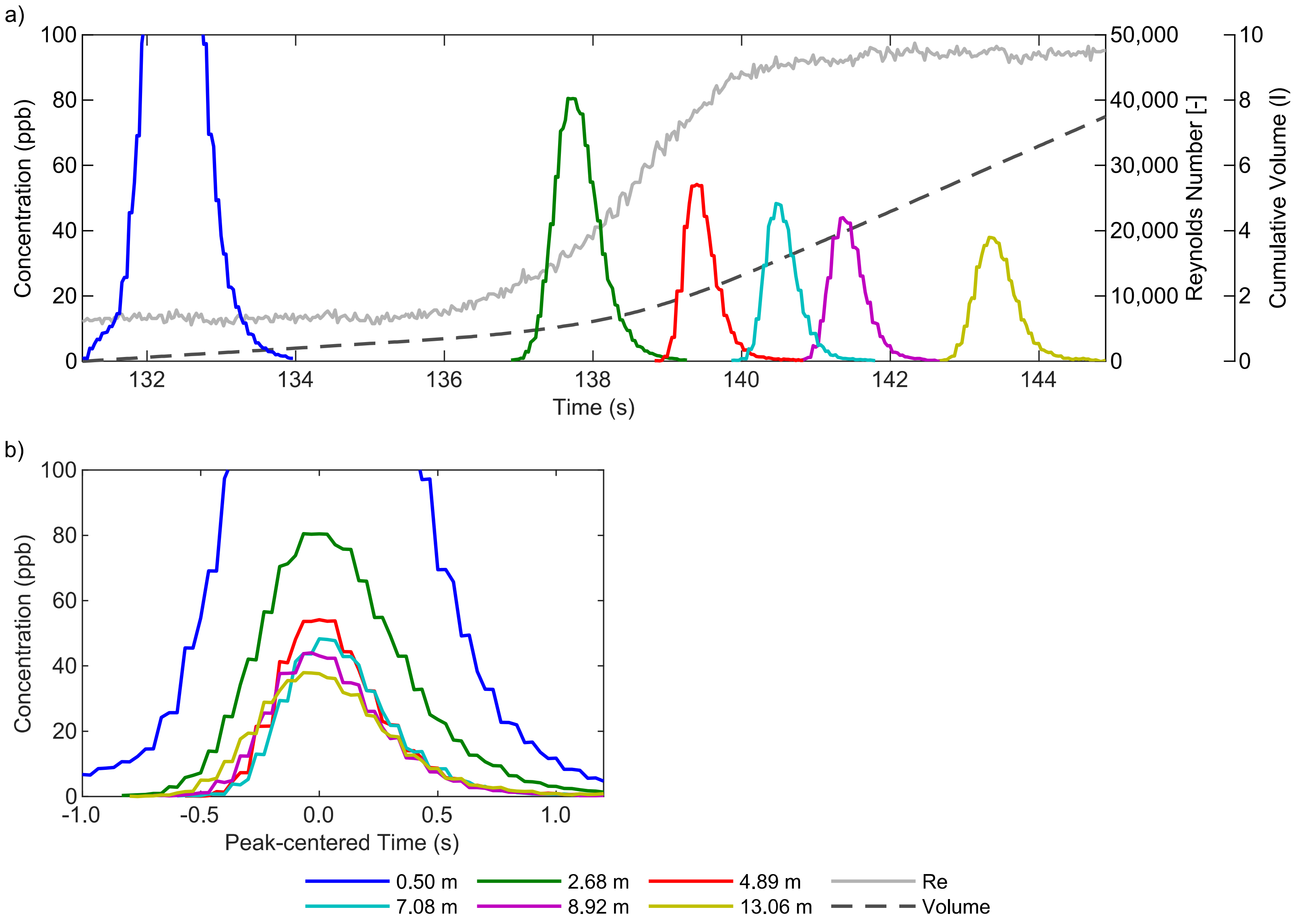
2. Taylor derived two equations for D_{xx}
    - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}



Unsteady Tests



Unsteady Tests



Werner & Kadlec (1996) :

'Flow weighted time':

$$\boxed{\phi(t) = V^{-1}\int_0^t Q(t)\mathrm{d}t}$$

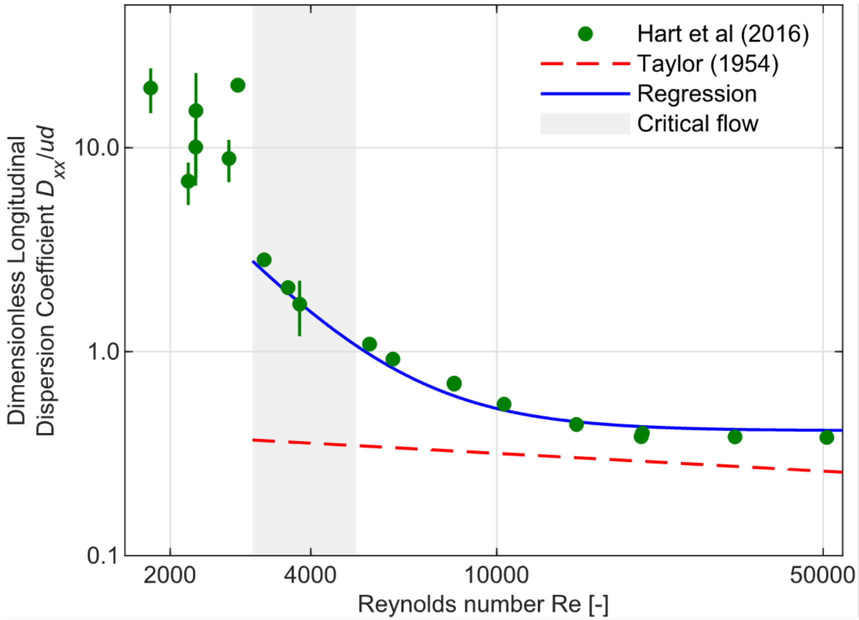
Unsteady Tests

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law

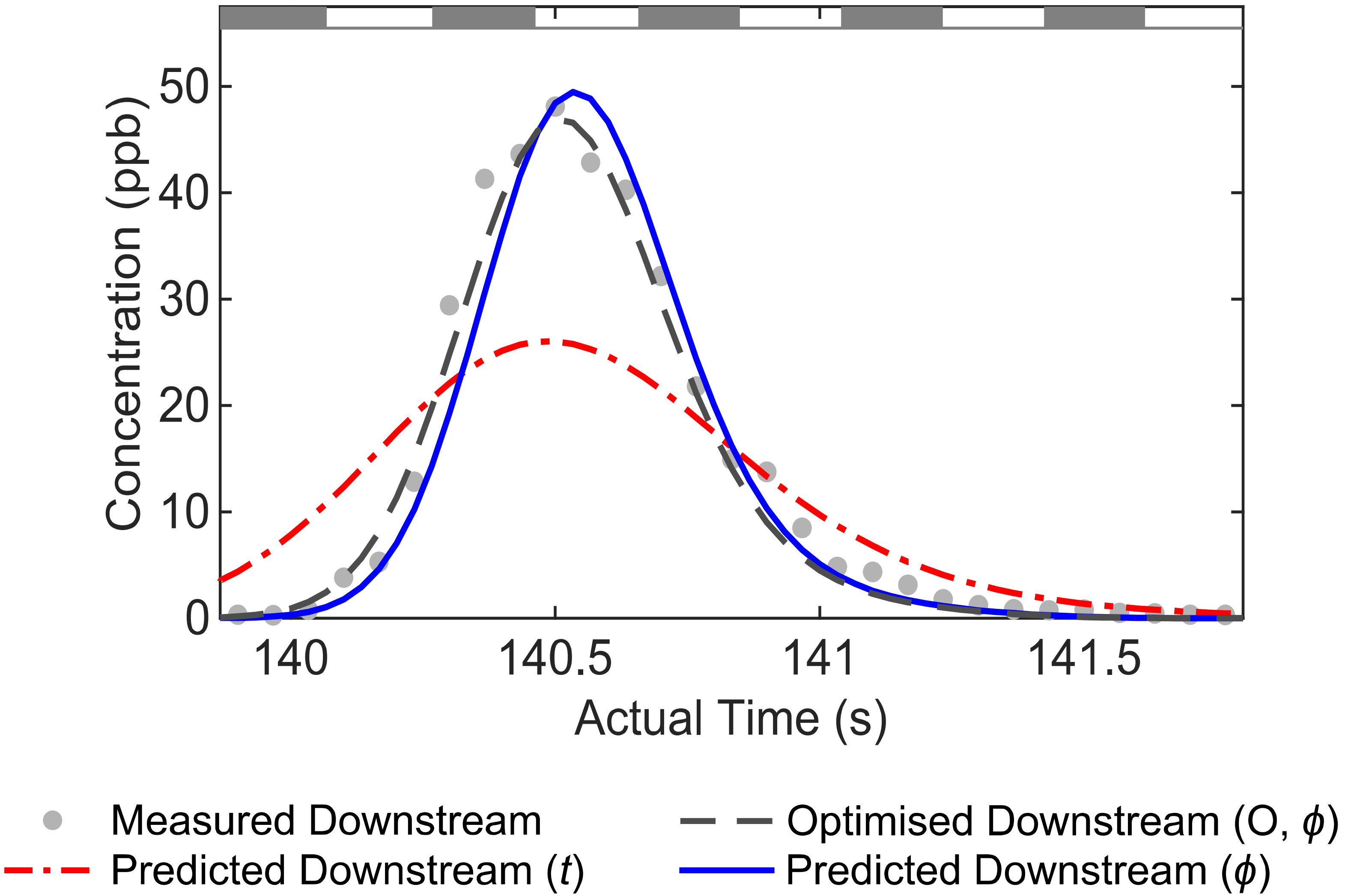
   - **Turbulent flow:** Gaussian transfer function is reasonable
   - **Laminar Flow:** At small times from injection, assume no radial mixing
   - **Unsteady flow:** Replace time with 'flow weighted time'

2. Taylor derived two equations for D_{xx}

   - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}
   - **Turbulent/Turbulent:** Use mean, 'as steady' $D_{xx} = f(Re)$



Unsteady Tests: Turbulent to Turbulent



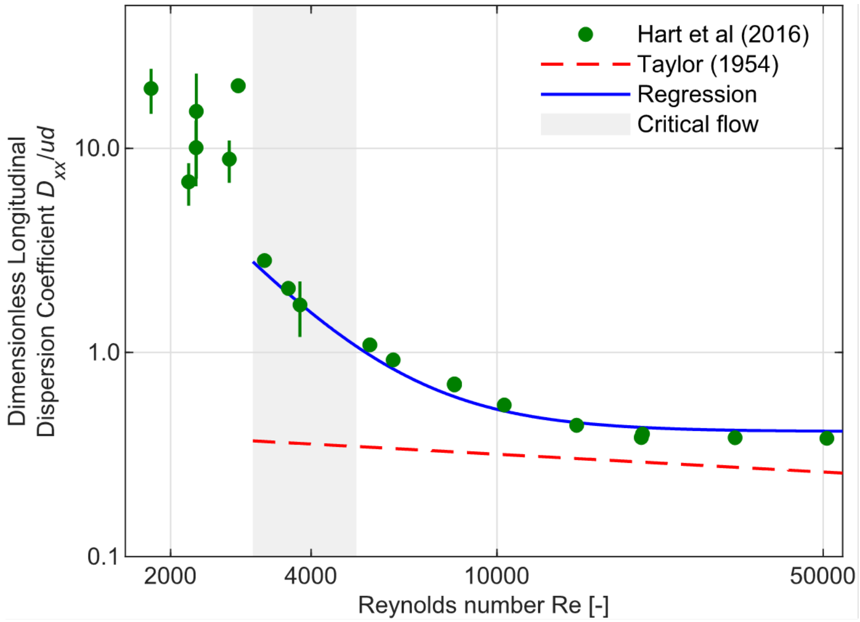
Unsteady Tests: Turbulent to Turbulent

1. Concentration profiles are Gaussian <u>(a certain time after injection)</u> ✓ and described by Fick's second

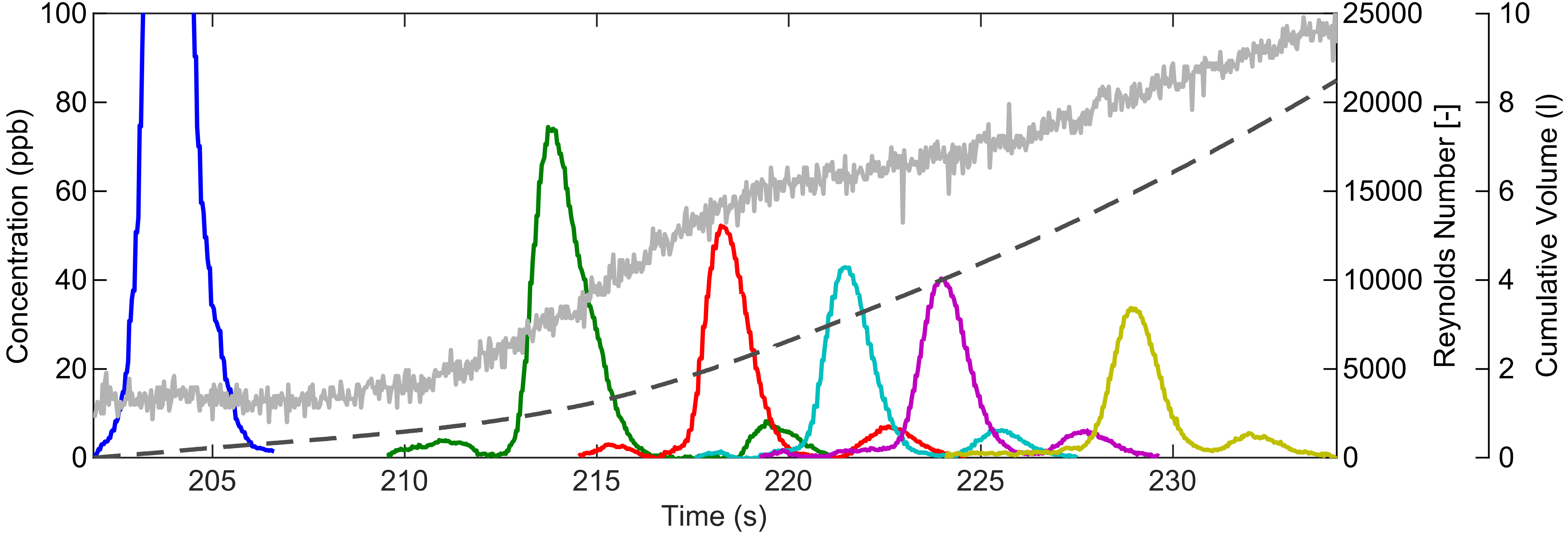
    - **Turbulent flow:** Gaussian transfer function is reasonable
    - **Laminar Flow:** At small times from injection, assume no radial mixing
    - **Unsteady flow:** Replace time with 'flow weighted time'
    - **Turbulent/Turbulent:** Gaussian transfer function is reasonable

2. Taylor derived two equations for D_{xx} ✓

    - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}
    - **Turbulent/Turbulent:** Use mean, 'as steady' $D_{xx} = f(Re)$



Unsteady Tests: Laminar to Turbulent

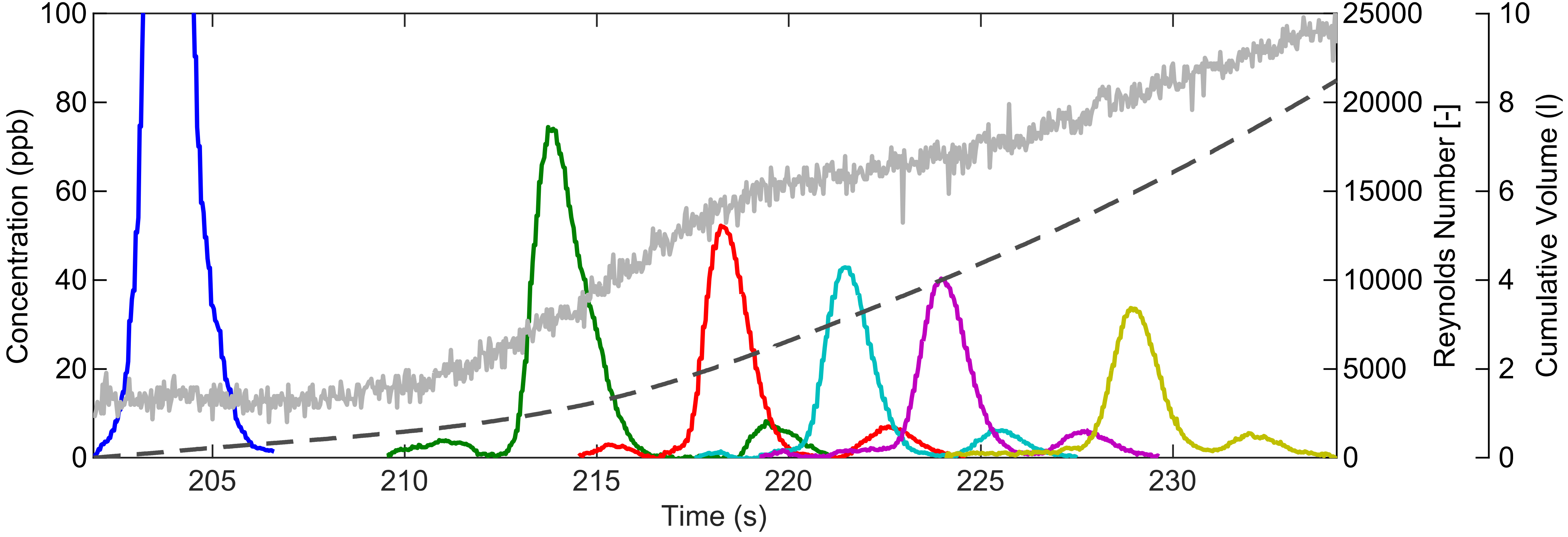


The story so far…

1. Concentration profiles are Gaussian (a certain time after injection) and described by Fick's second law
   - **Turbulent flow:** Gaussian transfer function is always reasonable
   - **Laminar Flow:** At small times from injection, assume no radial mixing
   - **Unsteady flow:** Replace time with 'flow weighted time'
   - **Turbulent/turbulent:** Flow, profiles are Gaussian
   - **Laminar/Turbulent:** flow profiles are disaggregated/NOT Gaussian

2. Taylor derived two equations for D_{xx}
   - **2000 < Re < 20,000:** Use Hart et al. (2016) for D_{xx}
   - **Turbulent/Turbulent:** Use mean, 'as steady' $D_{xx} = f(Re)$
   - **Laminar/Turbulent: ???**

Laminar to Turbulent Accelerating Flow



What is going on?