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Mixing Processes in Pipes, Sewers and the Natural Environment from Theory to Practice

Transverse dispersion in a compound channel

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Why is this work important?

Tewkesbury, UK November 2019

(GloucestershireLive.co.uk)

Overbank flow of the River Severn at Guarlford, November 2012

(DailyMail.co.uk)

Lateral transport processes can’t currently be forecast with certainty

Aim:

- To determine the lateral variation of transverse dispersion coefficient in a compound channel

Method:

- Laboratory experiments
- Dye tracing with continuous fluorescent tracer point source in steady overbank flow
- Optimisation using FDM & the depth averaged Advection Diffusion Equation

Outcome:

- Unique lateral variation of the transverse dispersion coefficient

Experimental apparatus:
Symmetrical compound channel

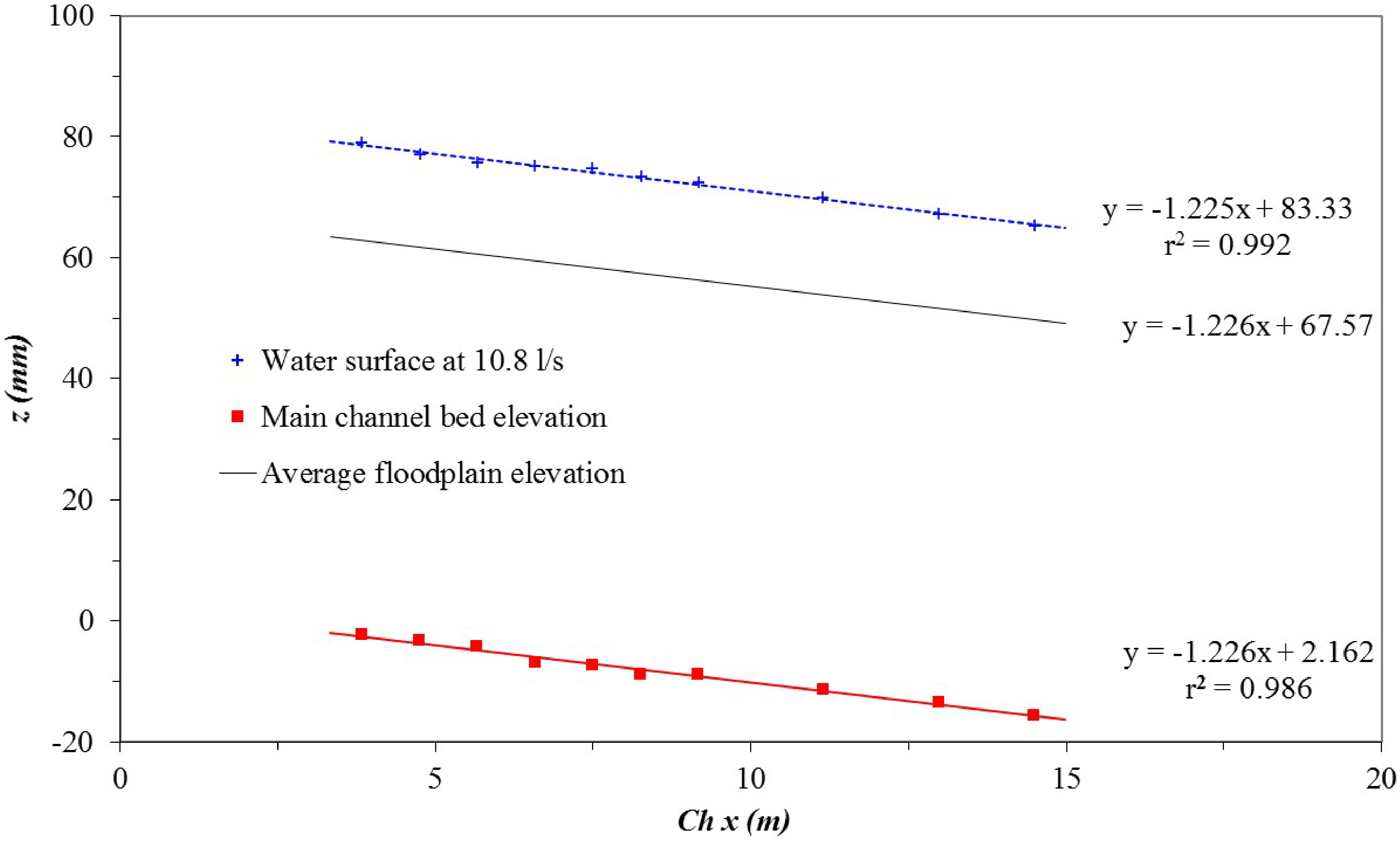
Constant head, pumped recirculating system

Dye tracing

constant head of Rhodamine WT

Concentrations extracted through small bore tubes to Turner Series 10 fluorometers via downstream peristaltic pump

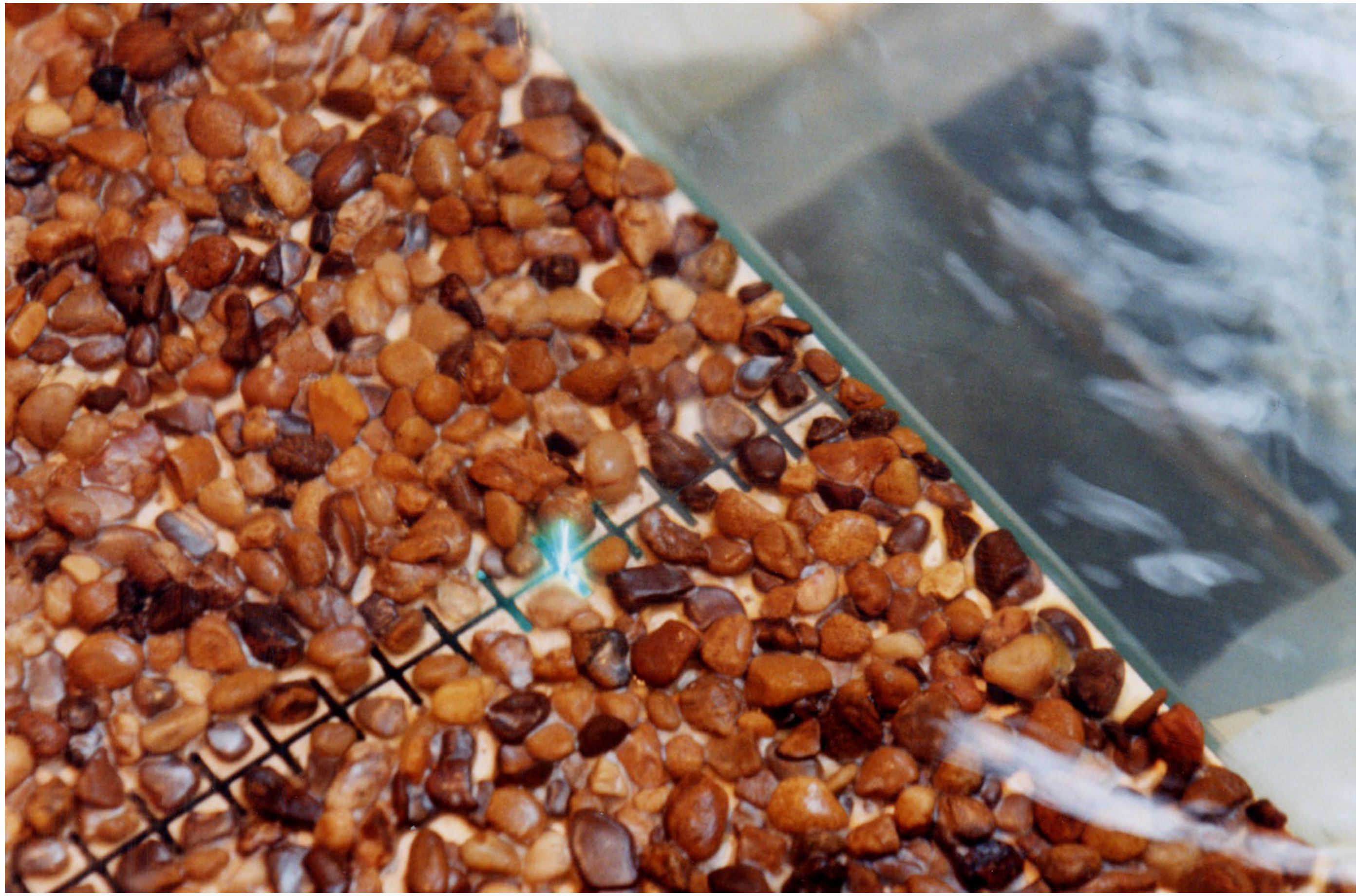
Uniform flow of 10.8 l/s, bed slope = 1 in 816, floodplain depth / main channel depth = 0.195



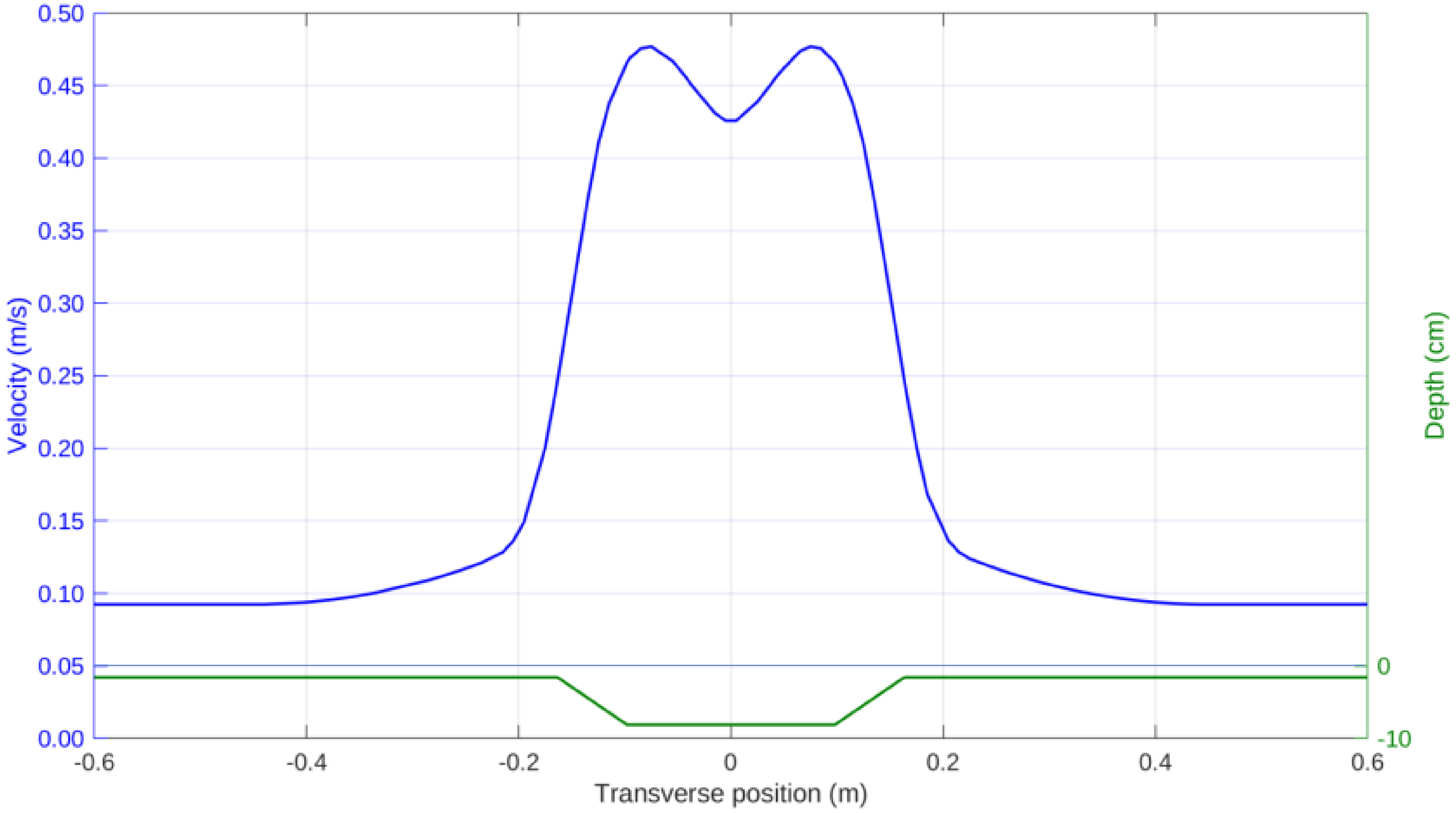
Experimental apparatus: symmetrical compound channel

Measurement section:

2D LDA for longitudinal and traverse velocities



Depth and depth mean longitudinal velocity (10.8L/s; H_R = 0.195)



Available overbank lateral concentration distributions

1. Main channel
2. Compound channel (i.e. main channel region and floodplain)
3. Floodplain

- Data had to be cleaned
  - Transverse spacing sometimes nonuniform; data interpolation a 1mm grid.
  - data scaled to ensure constant mass flux
    - impact in main channel region due to vertical mixing,
    - fluorometer sensitivity settings,
    - changes in dye source concentration

Method

- Apply Advection Diffusion Equation to upstream C vs y

$$u(y)\frac{\partial c(x,y)}{\partial x} = \frac{\partial}{\partial y}\left[D_y(y)\frac{\partial c(x,y)}{\partial x}\right]$$

Depth-averaged 2D ADE

Finite difference model formulation, allowing for variable *D*, *u*, and *h*

$$\frac{u^j\left[c_i^j - c_{i-1}^j\right]}{\Delta x} = \frac{1}{\Delta y}\left\{\frac{\left(D^{j+1} + D^j\right)}{2}\frac{\left[c_i^{j+1} - c_i^j\right]}{\Delta y} - \frac{\left(D^j + D^{j-1}\right)}{2}\frac{\left[c_i^j - c_i^{j-1}\right]}{\Delta y}\right\}$$

- FDM process outlined in West et al. (2021) appendix

Method

- Apply Advection Diffusion Equation to upstream C vs y

$$h(y)u(y)\frac{\partial c(x,y)}{\partial x}=\frac{\partial}{\partial y}\left[h(y)D_y(y)\frac{\partial c(x,y)}{\partial y}\right]$$

- and in Finite Difference form

$$\frac{h^j u^j\left[c_i^j - c_{i-1}^j\right]}{\Delta x}=\frac{1}{\Delta y}\left\{\frac{\left(h^{j+1}D^{j+1}+h^j D^j\right)}{2}\frac{\left[c_i^{j+1}-c_i^j\right]}{\Delta y}-\frac{\left(h^j D^j+h^{j-1}D^{j-1}\right)}{2}\frac{\left[c_i^j-c_i^{j-1}\right]}{\Delta y}\right\}$$

- FDM process outlined in West et al. (2021) appendix

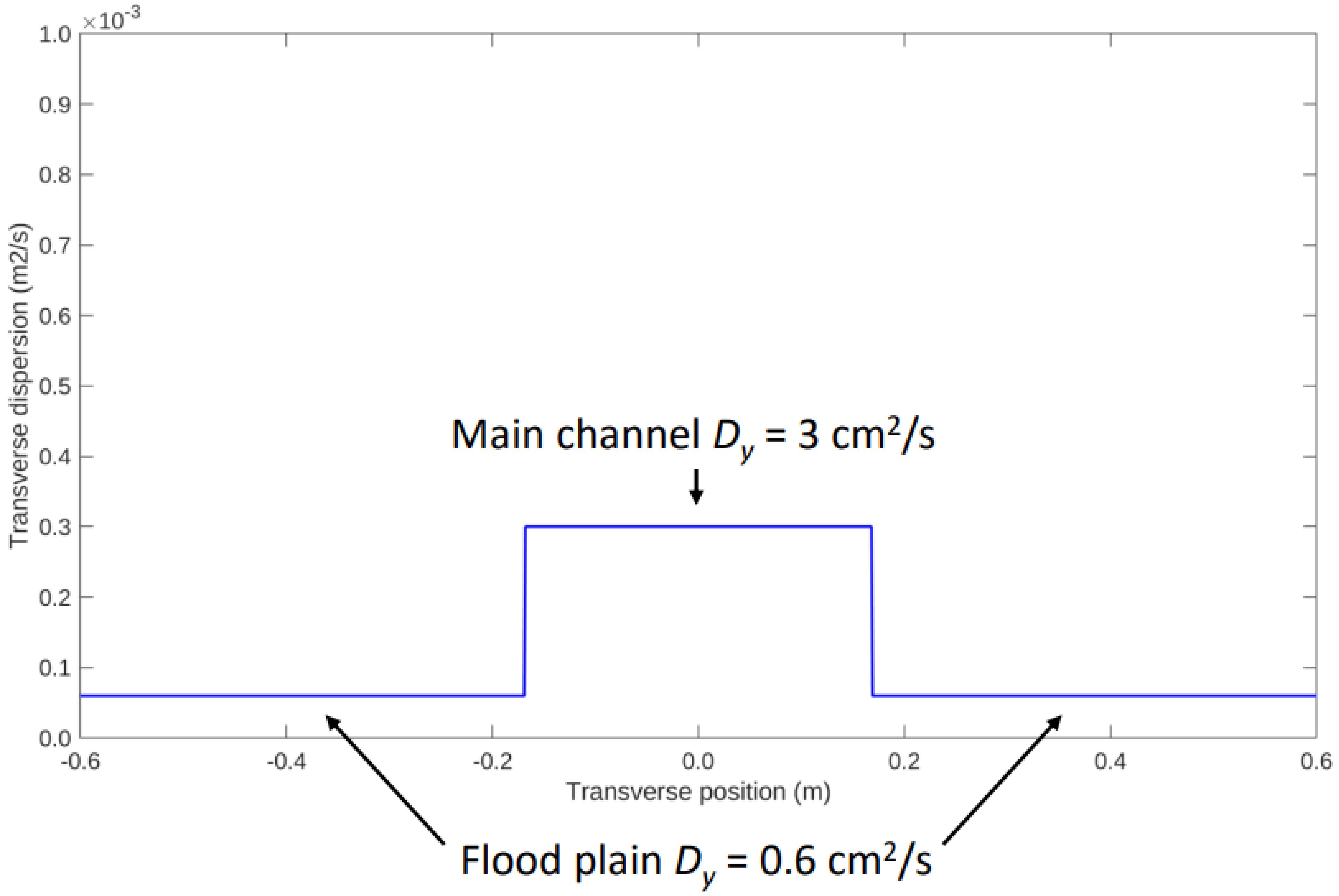
Method

- Comparison of measured & predicted concentrations C at a downstream reach using a goodness of fit R_t^2 (Young et al., 1980):

$$R_t^2 = 1 - \left(\sum (C - C_p)^2 / \sum C^2 \right)$$

- Lateral variation of D_y identified by optimising R_t^2

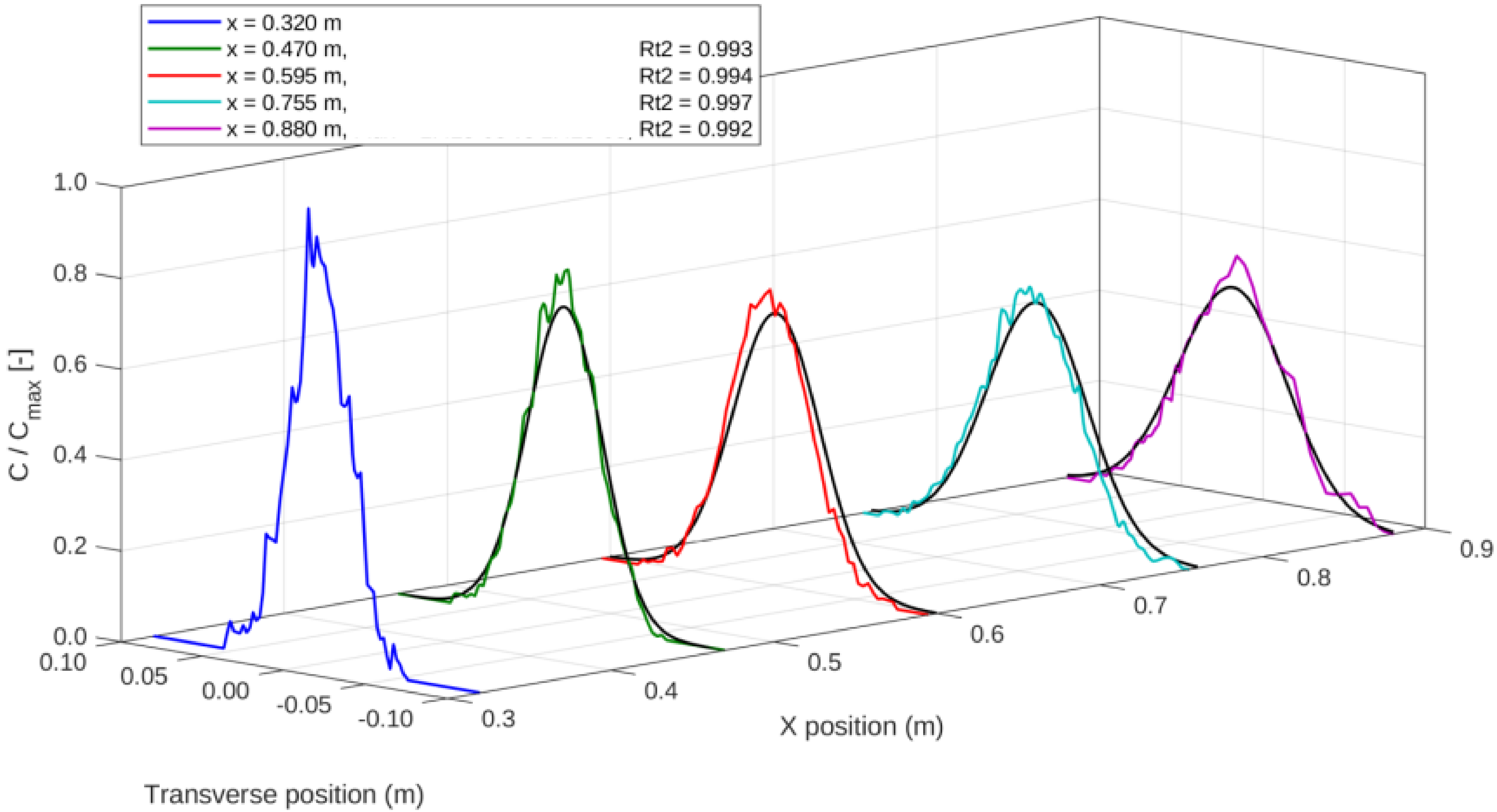
Choose the form of the lateral variation of D_y To explore in the beginning ..... a Step profile:



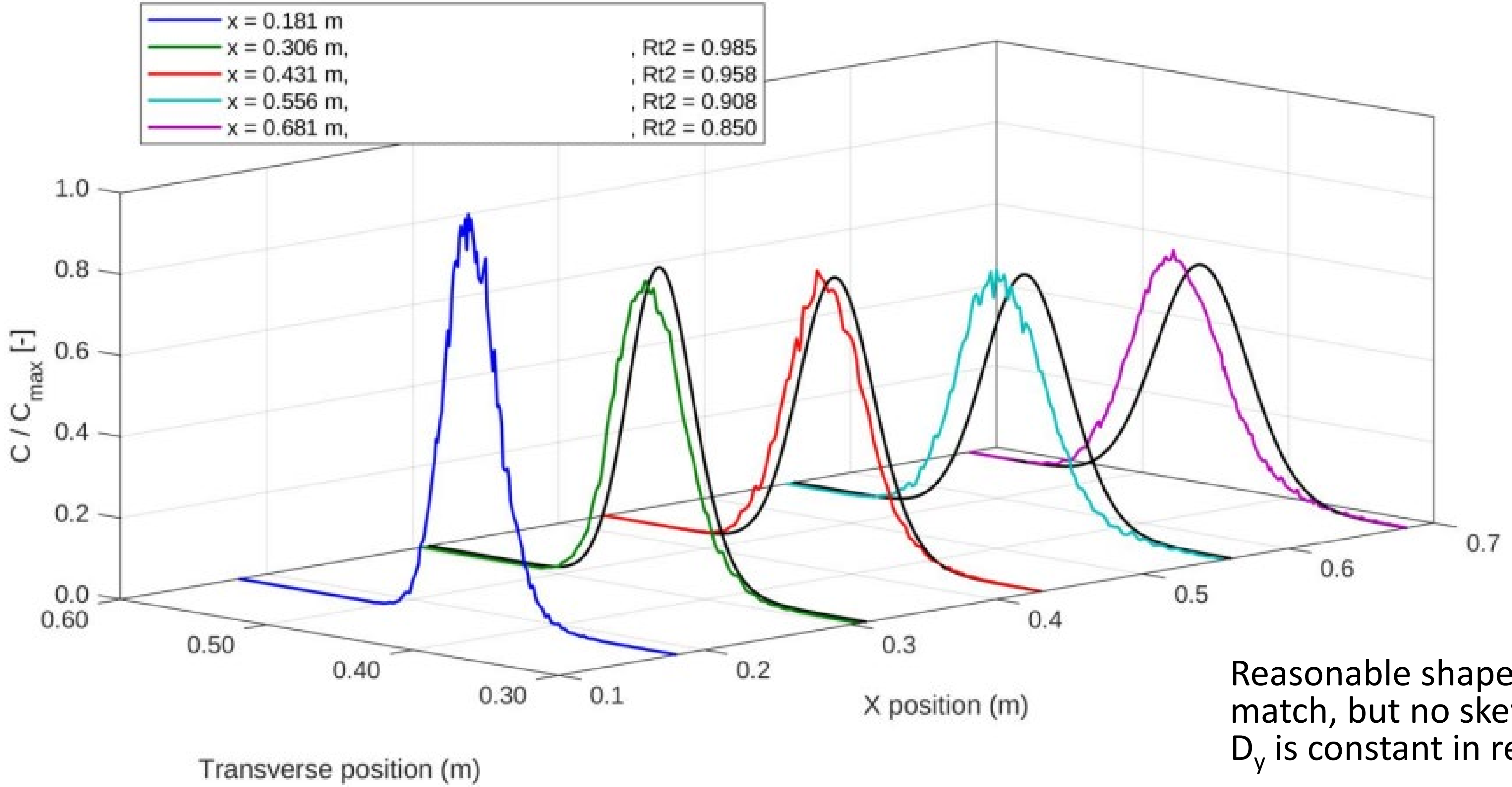
Interface fixed at the top of the bank, $x = \pm\ 0.168$ m.

D_y selected manually

FDM – Step D_y – Main channel

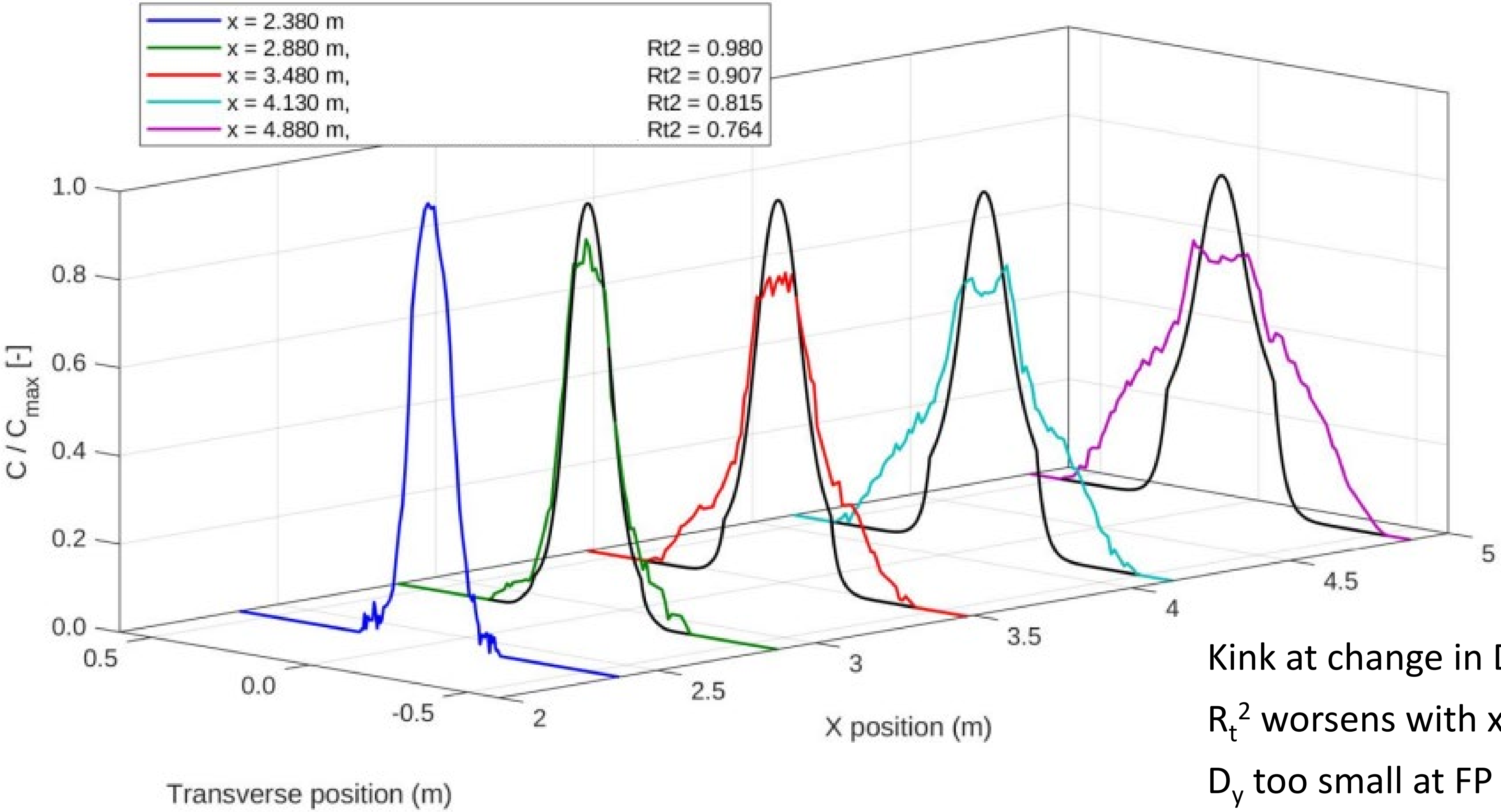


FDM – Step D_y – Floodplain



Reasonable shape match, but no skew as D_y is constant in region

FDM – Step D_y – Main channel & Floodplains

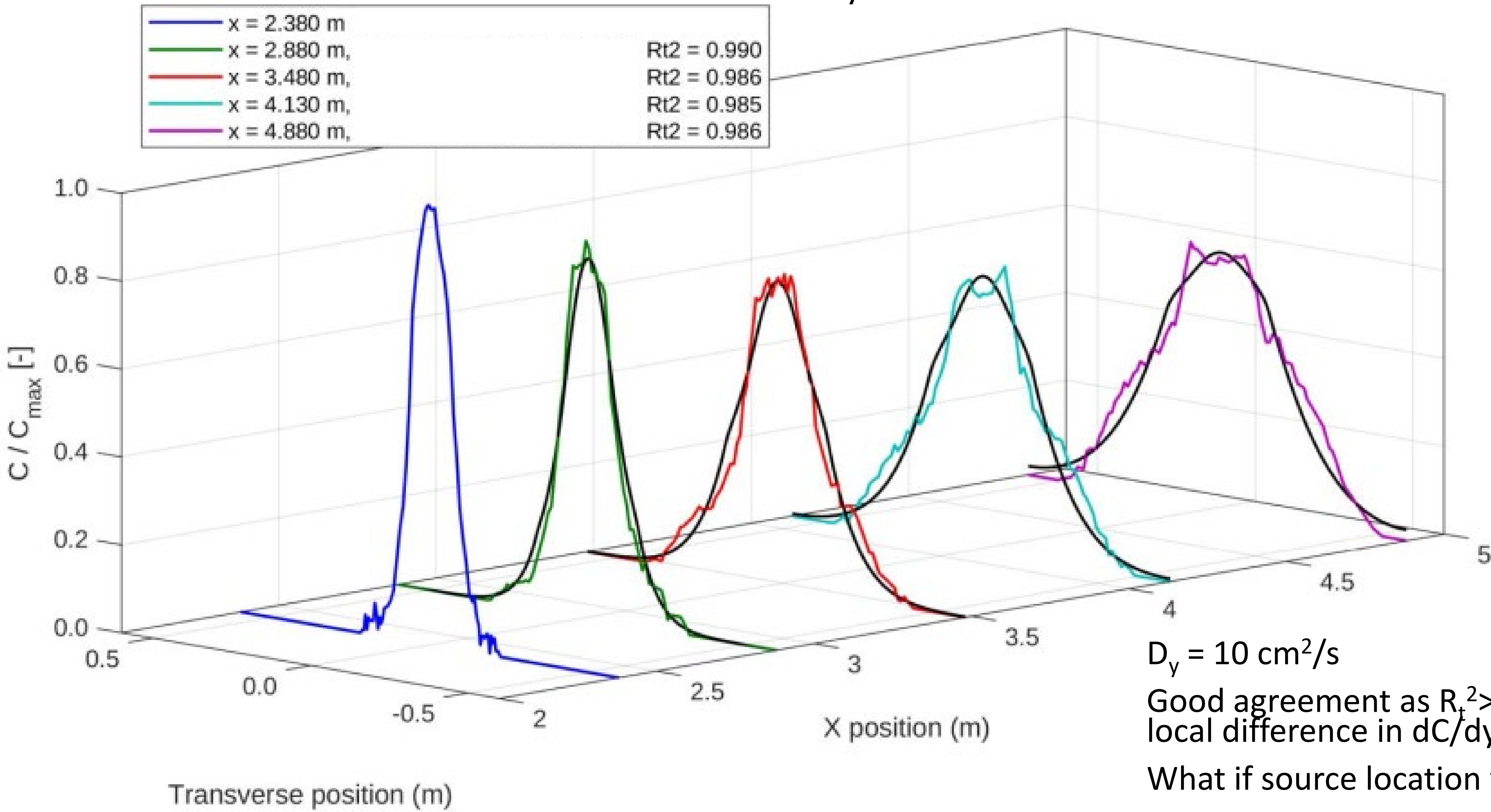


Kink at change in D_y

R_t^2 worsens with x

D_y too small at FP start

Possible to use a uniform D_y – Main channel & Floodplains



D_y = 10 cm^2/s

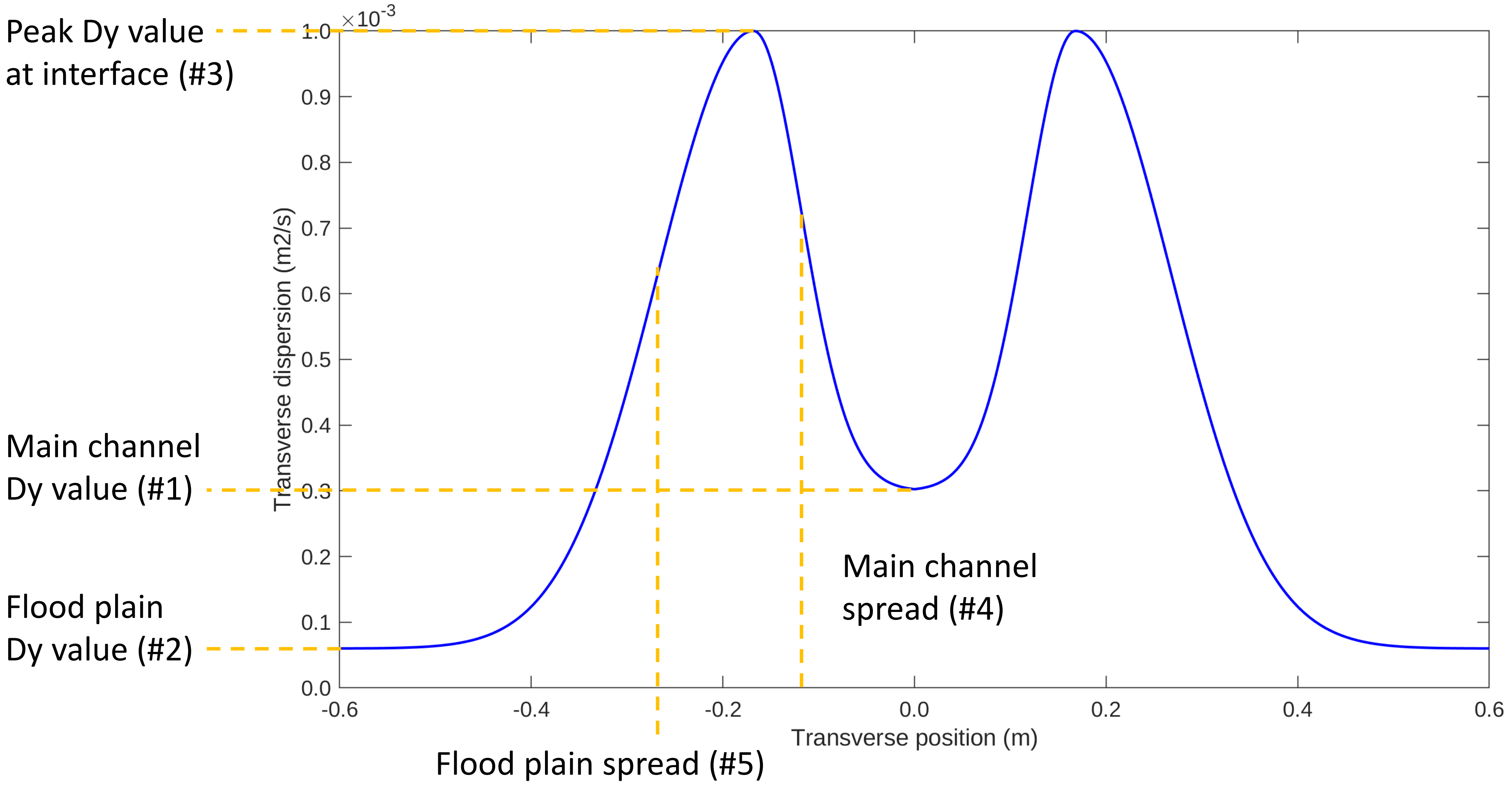
Good agreement as $R_t^2 > 0.98$ but local difference in dC/dy

What if source location varies?

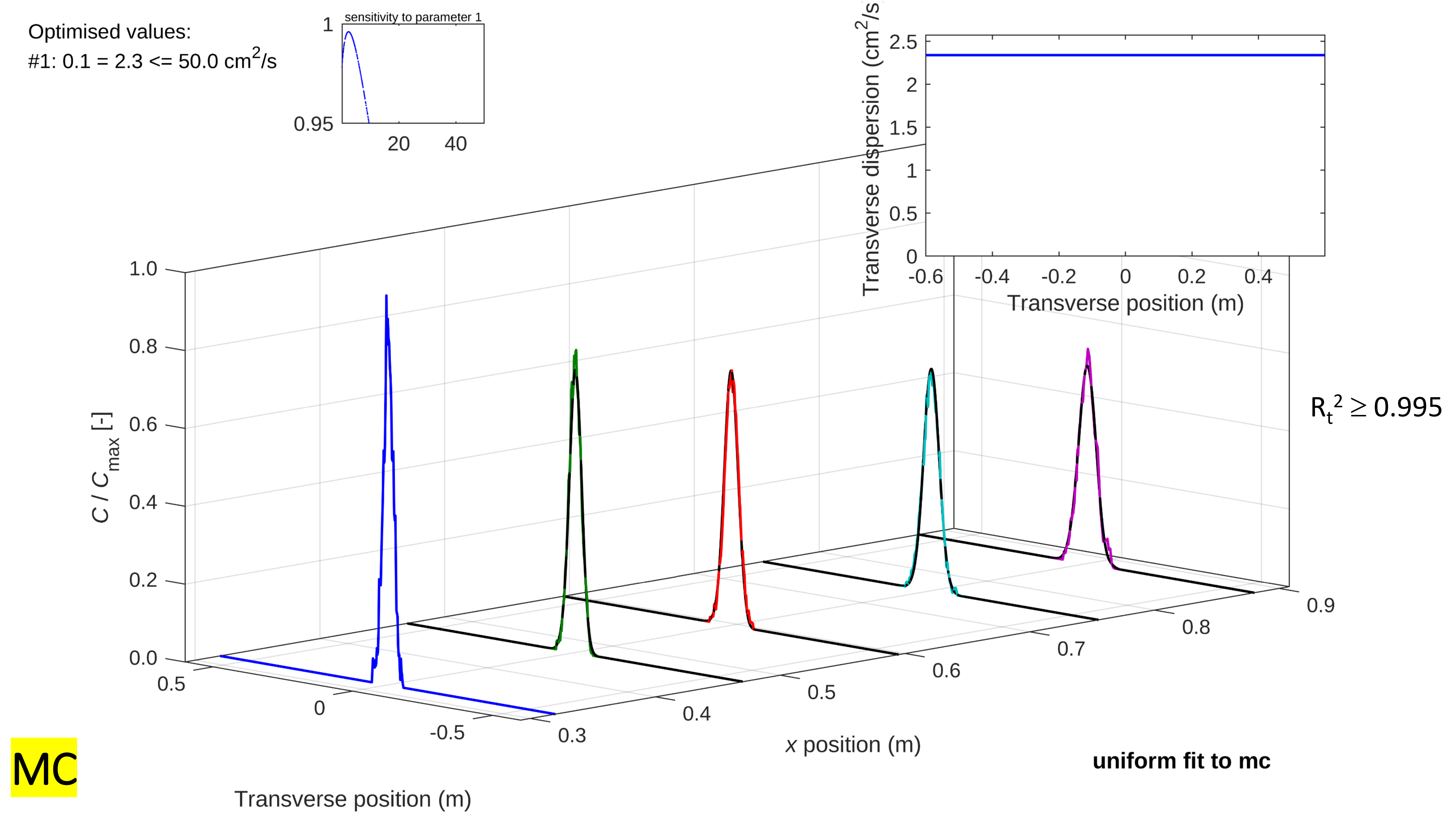
FDM Optimisation Results

Dual semi-Gaussian dispersion coefficient

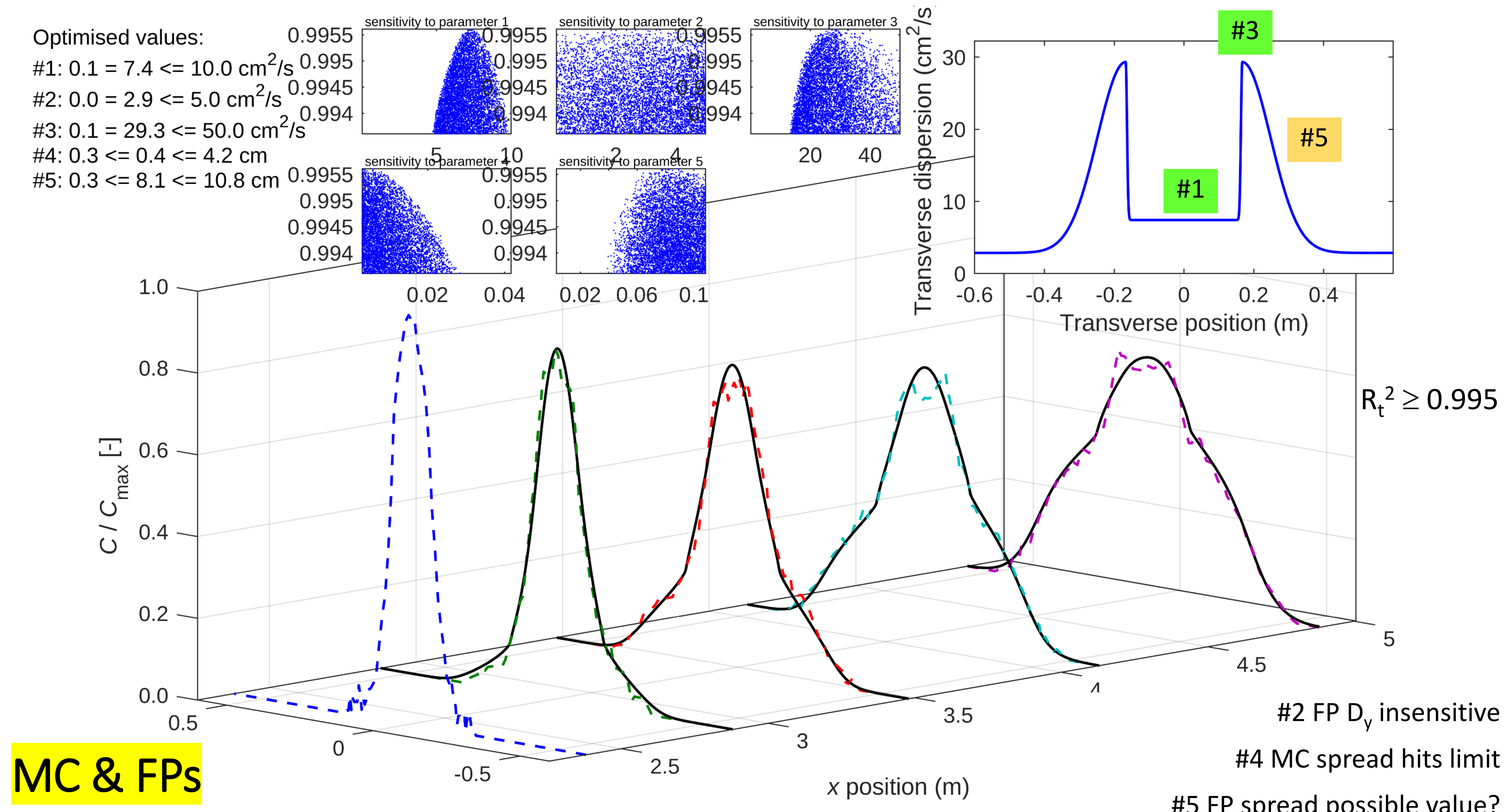
Note when doing a 4 parameter optimisation with a fixed main channel value, the other parameter numbers decrease by 1

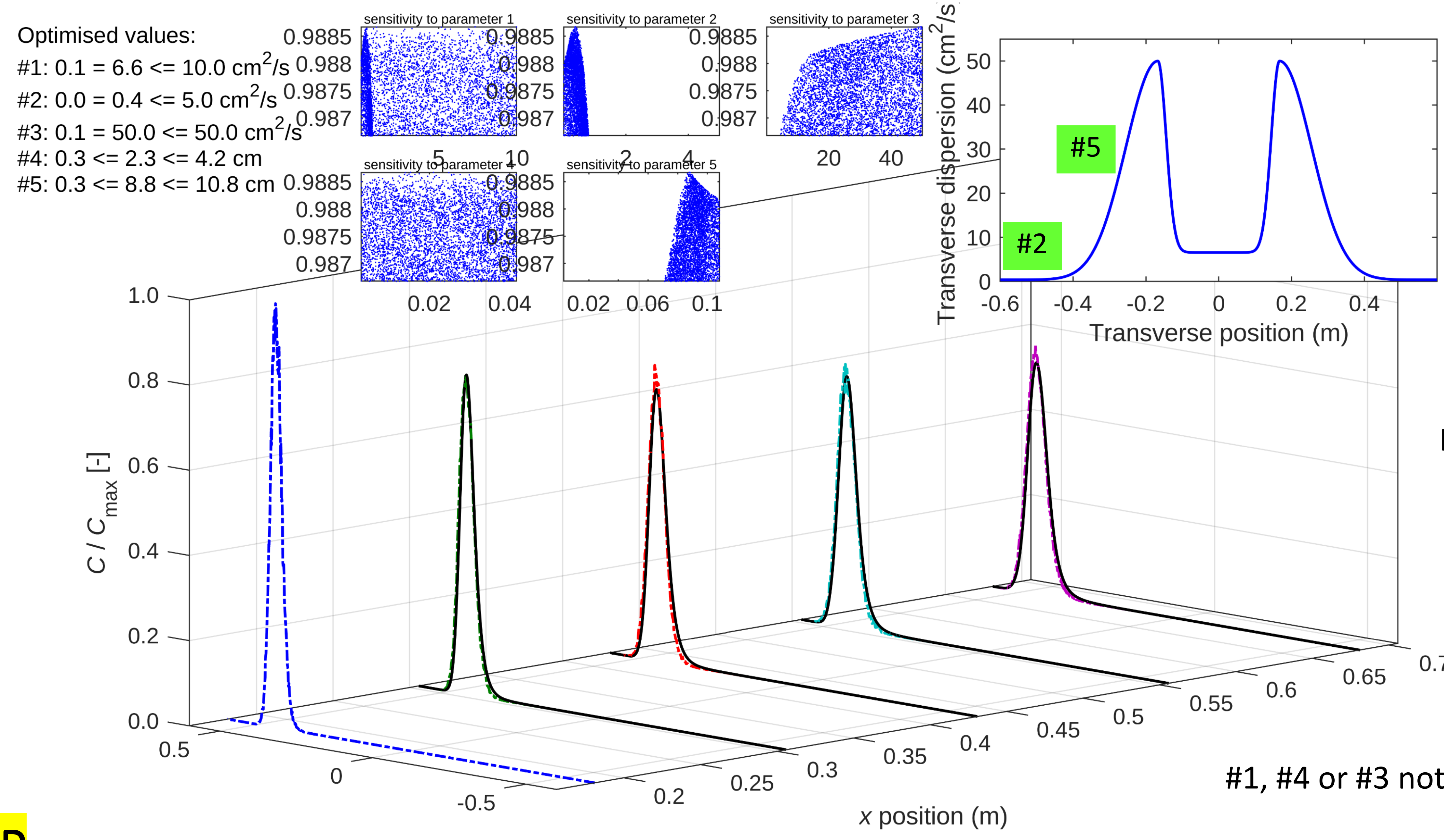


Spreads are standard deviation



Optimised values:
#1: 0.1 = 2.3 <= 50.0 cm^2/s
sensitivity to parameter 1
Transverse dispersion (cm^2/s)
Transverse position (m)
C / C$_{max}$ [-]
x position (m)
Transverse position (m)
$R_t^2 \geq 0.995$
uniform fit to mc
MC


Optimised values:
#1: 0.1 = 7.4 <= 10.0 cm²/s
#2: 0.0 = 2.9 <= 5.0 cm²/s
#3: 0.1 = 29.3 <= 50.0 cm²/s
#4: 0.3 <= 0.4 <= 4.2 cm
#5: 0.3 <= 8.1 <= 10.8 cm
sensitivity to parameter 1
sensitivity to parameter 2
sensitivity to parameter 3
sensitivity to parameter 4
sensitivity to parameter 5
Transverse dispersion (cm²/s)
Transverse position (m)
#1
#3
#5
C / C_max [-]
x position (m)
R_t² ≥ 0.995
#2 FP D_y insensitive
#4 MC spread hits limit
#5 FP spread possible value?
MC & FPs



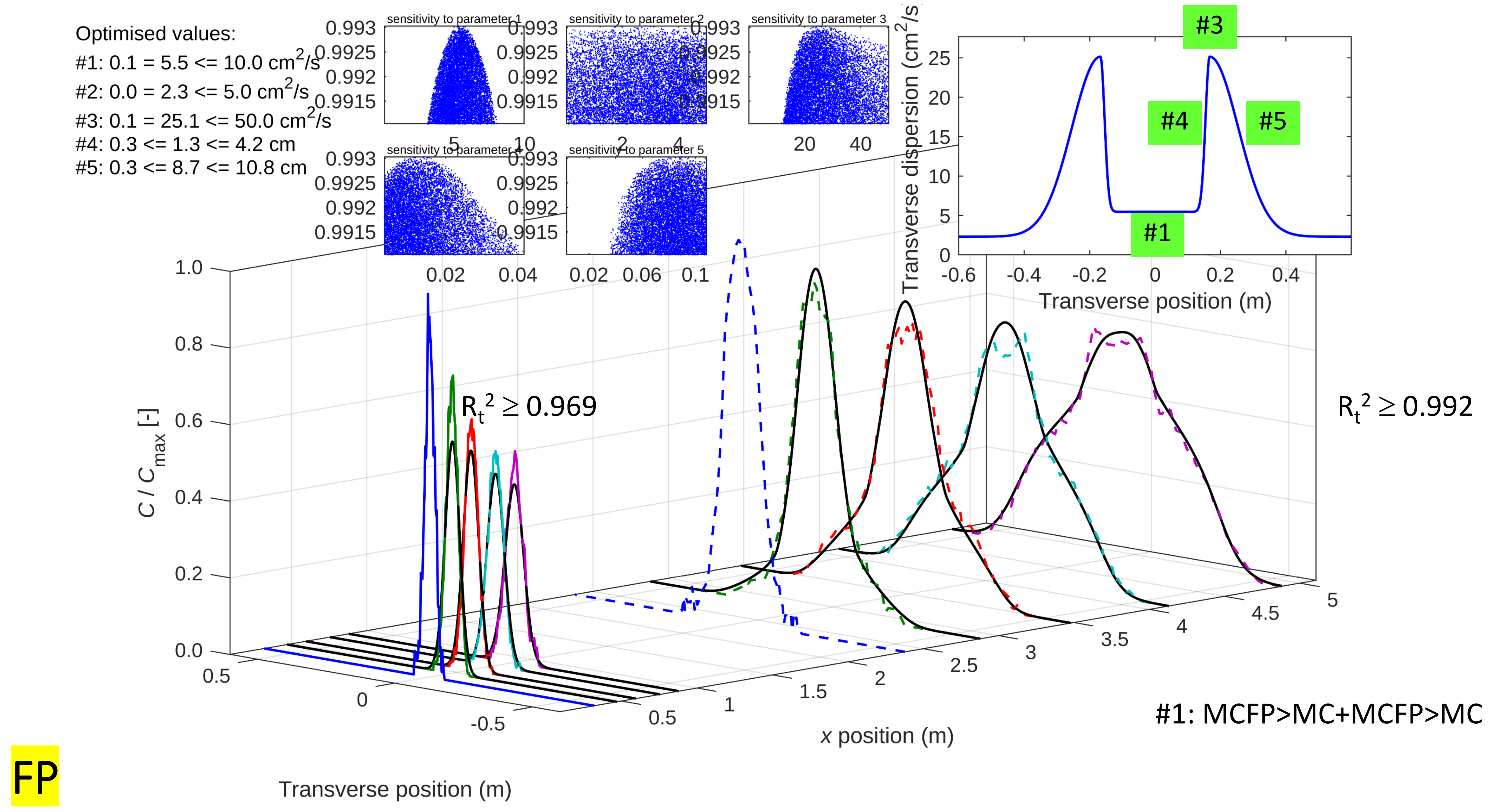
$R_t^2 \geq 0.995$

$R_t^2 \geq 0.995$

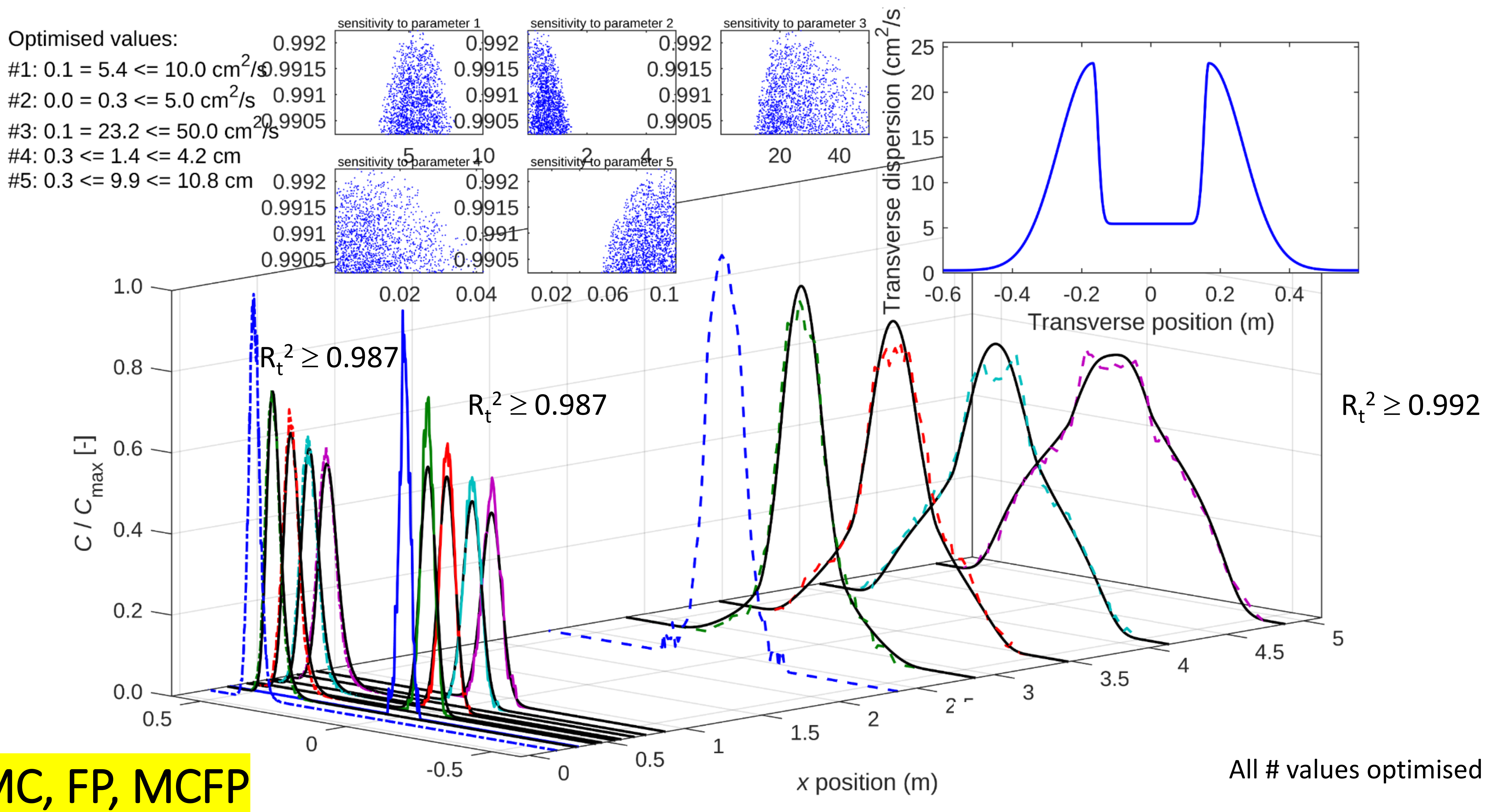
#1, #4 or #3 not optimised

FP

Transverse position (m)



Optimised values:
#1: 0.1 = 5.5 <= 10.0 cm^2/s
#2: 0.0 = 2.3 <= 5.0 cm^2/s
#3: 0.1 = 25.1 <= 50.0 cm^2/s
#4: 0.3 <= 1.3 <= 4.2 cm
#5: 0.3 <= 8.7 <= 10.8 cm
sensitivity to parameter 1
sensitivity to parameter 2
sensitivity to parameter 3
sensitivity to parameter 4
sensitivity to parameter 5
#3
#4
#5
#1
Transverse dispersion (cm^2/s)
Transverse position (m)
$R_t^2 \geq 0.969$
$R_t^2 \geq 0.992$
C / C_max [-]
x position (m)
#1: MCFP>MC+MCFP>MC
FP
Transverse position (m)


Optimised values:
#1: 0.1 = 5.4 <= 10.0 cm²/s
#2: 0.0 = 0.3 <= 5.0 cm²/s
#3: 0.1 = 23.2 <= 50.0 cm²/s
#4: 0.3 <= 1.4 <= 4.2 cm
#5: 0.3 <= 9.9 <= 10.8 cm
sensitivity to parameter 1
sensitivity to parameter 2
sensitivity to parameter 3
sensitivity to parameter 4
sensitivity to parameter 5
Transverse dispersion (cm²/s)
Transverse position (m)
C / C_max [-]
x position (m)
R_t² ≥ 0.987
R_t² ≥ 0.987
R_t² ≥ 0.992
All # values optimised

MC, FP, MCFP

Summary of D_y

	#1 MC cm^2/s	#2 FP cm^2/s	#3 MC/FP cm^2/s	#4 σ_{MC} cm	#5 σ_{FP} cm	R_t^2 MC	R_t^2 MCFP	R_t^2 FP
DSG MC optimised	2.3					> 0.995		
DSG MCFP optimised	7.4	x	29.3	x	8.1		> 0.995	
DSG FP optimised	x	0.4	x	x	8.8		> 0.995	
DSG MC & MCFP optimised	5.5	x	25.1	1.3	8.7	> 0.969	> 0.992	
DSG MC & MCFP & FP optimised	5.4	0.3	23.2	1.4	9.9	0.987	0.987	0.987

Conclusions

- Normalised #1 Main channel (D_y/Hu_*) = 0.09 to 0.13 (depending on u_* calc)
    - = 0.42 with MC bank reflections
- Normalised #2 (D_y/Hu_*) = 0.18 over the rough floodplain

- Peak D_y at start of floodplain x4 main channel D_y (x77 floodplain D_y)

- Spread onto FP (#5) 7x spread into MC (#4).
- Spread can be identified without values of Peak or tail D_y – depends on shape?

- Dual Semi Gaussian distribution works, but optimisation of all five # factors required 3 tracer sources

References

Guymer, I., & Spence, K. (2009). Laboratory study of transverse solute mixing during over-bank flows. In Proc. of 33rd IAHR Congress: Water Engineering for a Sustainable Environment.

Spence, K.J. and Guymer, I. (2021). Defining the distribution of transverse mixing coefficients across a rough laboratory flood plain. In: KALINOWSKA, Monika, (ed.) 6th IAHR Europe Congress Warsaw Poland 2021: Abstract book. IAHR, 319-320.

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