Nonlinear Scaling of Resource Allocation in Sensory Bottlenecks

Laura R. Edmondson[1,3], Alejandro Jiménez-Rodriguez[2,3], Hannes P. Saal[1,3]
(1) Active Touch Lab, Department of Psychology, University of Sheffield, UK
(2) Department of Computer Science, University of Sheffield, UK
(3) Sheffield Robotics, University of Sheffield, UK

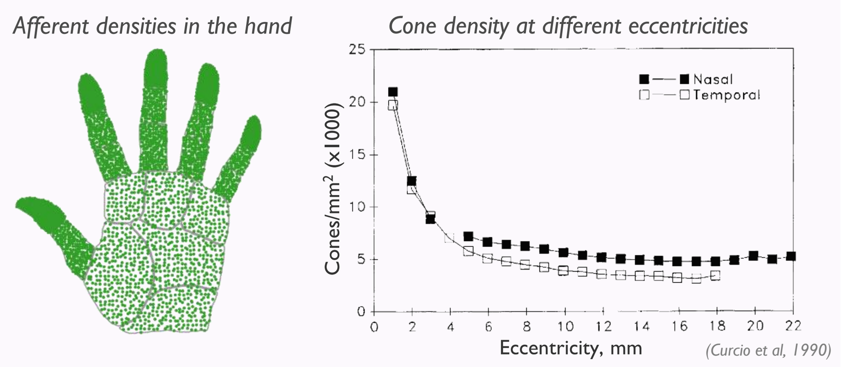
Background

I: Receptor densities

Densities of sensory receptors vary across space according to the sensory modality.

In the retina: greater number of cones in the fovea than the periphery.

In the skin: afferent density increases towards the fingertips.

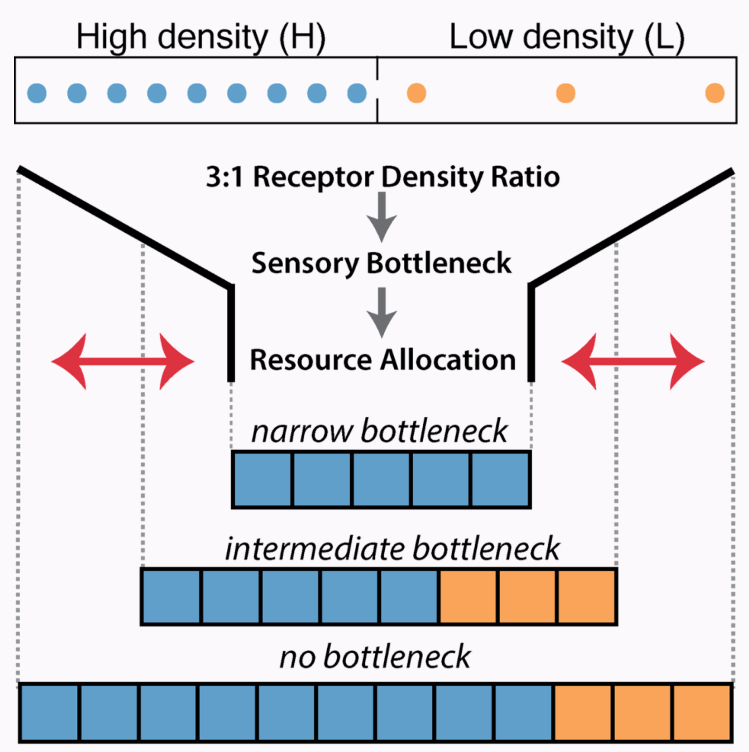


II: Bottleneck width

Bottlenecks, such as the optic nerve, can be of different sizes. The bottleneck width constrains the amount of information which can pass through to output neurons.

III: Resource allocation

Output neurons represent receptors in the high and low density region. However, it is unclear how many should be allocated to each region.

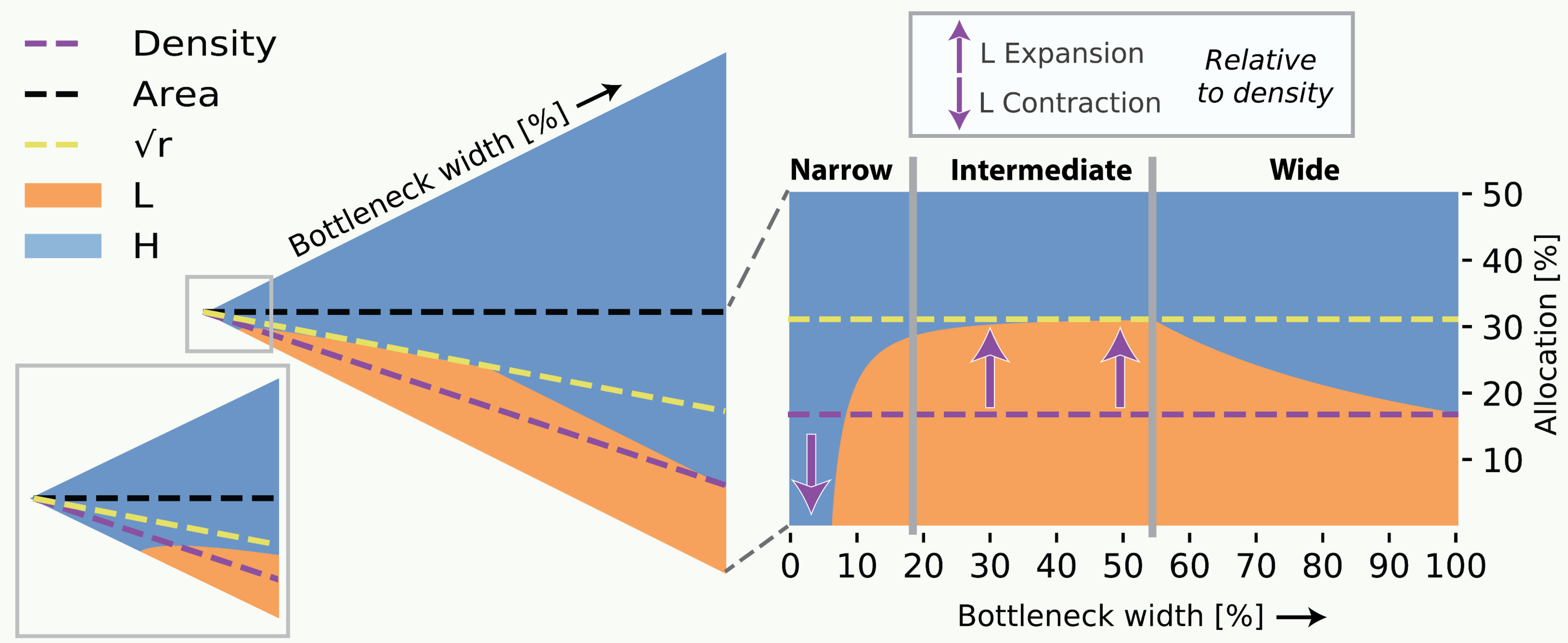


Efficient coding models

Efficient coding theory has been used to predict how the brain should allocate its limited resources through maximising information.
This can be achieved by reducing redundancy in sensory systems from correlations of input signals.

How should sensory information be allocated in processing bottlenecks, given different receptor input densities and bottleneck widths?

Results



Allocation strategy changes depending on the width of the bottleneck

Narrow

All or almost all output neurons are allocated to the high density input region, leading to an expansion of this region in the bottleneck.

Intermediate

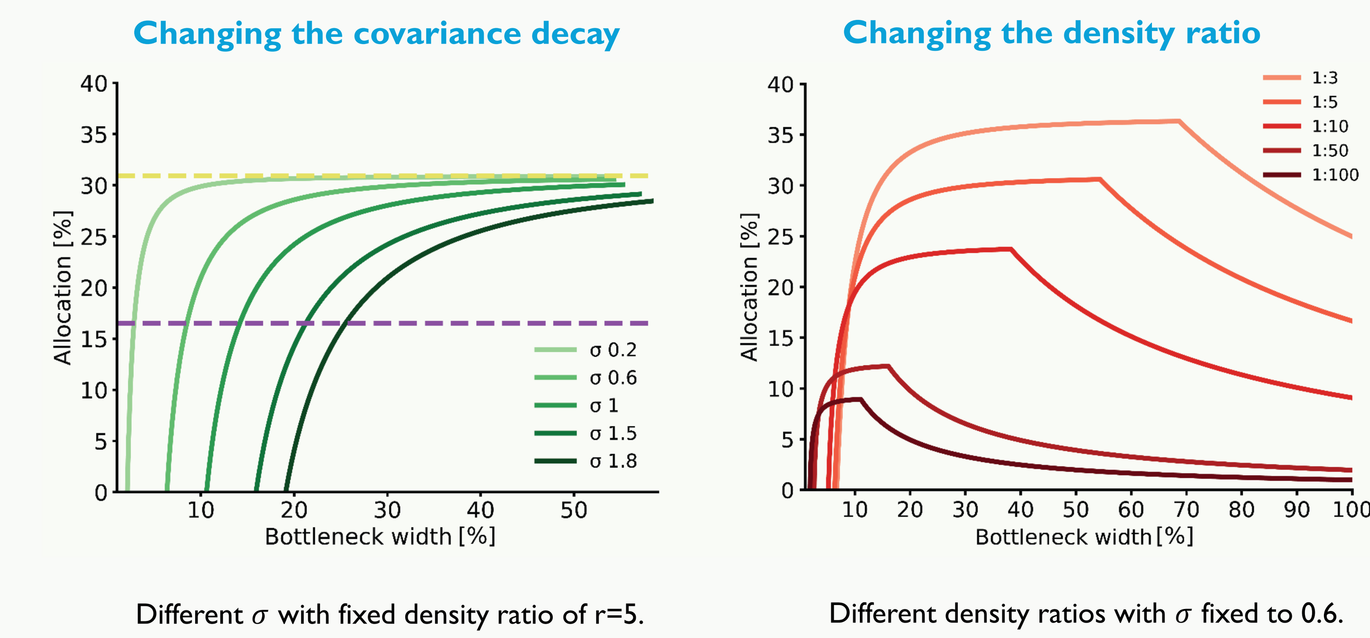
Output neurons are allocated at a ratio of √r to the high density region. Typically the high density region is underrepresented compared to its density.

Wide

Low density region has one output neuron assigned for each input, so all the information from this region is captured in the bottleneck. All additional output neurons are assigned to the high density region.

Results

Bottleneck resource allocation



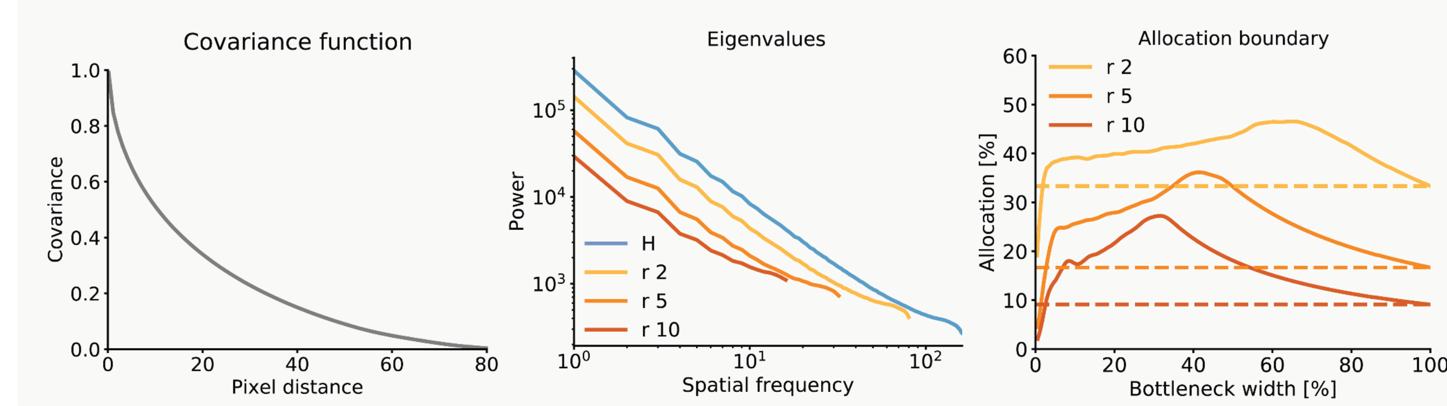
Different σ with fixed density ratio of r=5.

Different density ratios with σ fixed to 0.6.

Application to natural image statistics

How should neurons in a visual bottleneck be allocated, reflecting the non-uniform densities of cones across the retina?

- Natural images follow a power spectral density, where power decreases with $1/f^2$ for increasing spatial frequencies.
- Image data taken from the SALICON dataset.



Method

Whitening model of sensory information

- Whitening models maximise information through decorrelation.
- We can represent sensory information with fewer outputs by selecting fewer than the complete set of components from eigenvalue decomposition.
- Returned data should be decorrelated. Solutions are in the form:

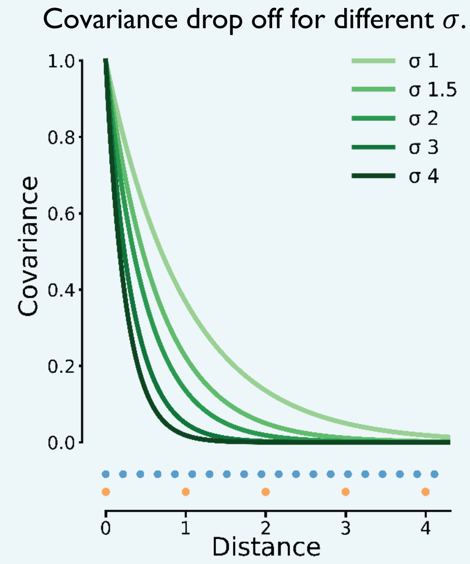
$$W = P\Lambda^{-\frac{1}{2}}U^T$$

Where Λ is a diagonal matrix containing the eigenvalues of the input covariance, and U contains its eigenvectors. P is any orthogonal matrix that yields required filters.

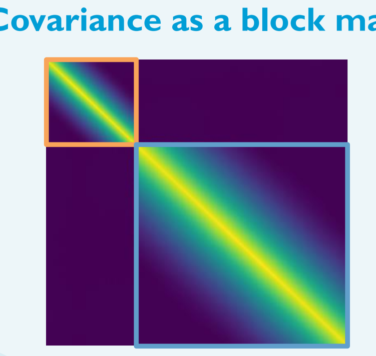
Block matrices and covariances of sensory inputs

Covariance of receptors

Covariance drop off for different σ.



Covariance as a block matrix



Block matrix approximation

The exponential covariance for receptor responses is:

$$\Sigma_{ij} = \exp(-\sigma|x_i - x_j|)$$

Where x_i is the location of the ith receptor and σ is the decay constant.

Block matrix approximation

If we approximate the two regions as non-overlapping using block diagonal matrices, the covariance is:

$$\Sigma = \begin{bmatrix} \Sigma_H & 0 \\ 0 & \Sigma_L \end{bmatrix}$$

By Cauchy Interlacing theorem, the eigenvalues of the block diagonal will be equivalent to the combined set of Σ_H and Σ_L:

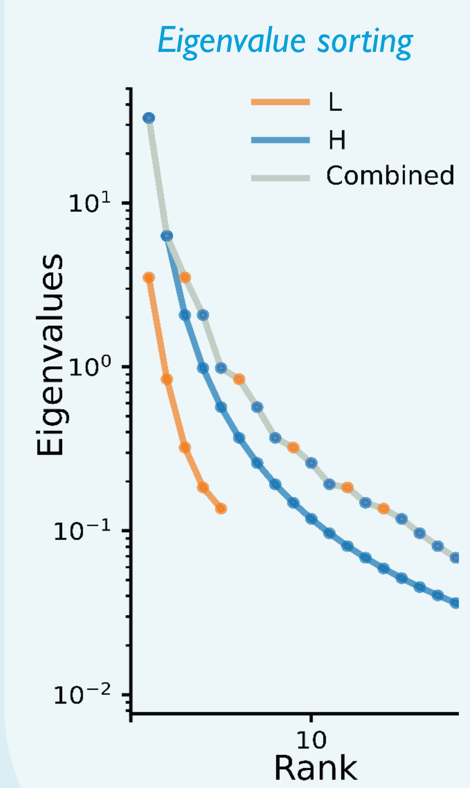
$$\Lambda = \begin{bmatrix} \Lambda_H & 0 \\ 0 & \Lambda_L \end{bmatrix} \quad U = \begin{bmatrix} U_H & 0 \\ 0 & U_L \end{bmatrix}$$

Where Λ is the Eigenvalues and U is the Eigenvectors.

Finding Eigenvalues analytically

For a sensory bottleneck with m output neurons, we retain only the m largest eigenvalues from Λ along with their associated components in U.

Eigenvalue sorting



Eigenvalues for the two regions

Region H:

$$\lambda_h = \frac{2\sigma}{h^2\pi^2S^{-2} + \sigma^2}$$

Region L:

$$\lambda_l = \frac{2\sigma}{l^2\pi^2S^{-2}r + \sigma^2 r}$$

Where $r > 1$ is the ratio of higher and lower densities, σ is the covariance decay, and S is the size of the low density region.

Calculating allocation in the bottleneck

For the block case, the number of output neurons allocated to each region under different bottleneck widths can be solved by ordering the eigenvalues from the decomposition of each region.

$$h = \frac{\sqrt{l^2\pi^2 r + S^2\sigma^2 r - S^2\sigma^2}}{\pi}$$

As more neurons are allocated to L, the ratio of allocated neurons simplifies to √r, where r is the ratio of high versus low density.

Summary and conclusions

- Limiting the number of output neurons does not lead to proportional representation of sensory receptors.
- Input region representation can contract and expand depending on the width of the bottleneck.
- Extent of spatial correlations and different ratios of receptor densities also influences the results.

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